MODELLING AND CONTROL OF A KIND OF PARALLEL MECHANISM DRIVEN BY PIEZOELECTRIC ACTUATORS

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Abstract

We describe a micro-operating mechanism for a human-scale tele-operating system. The Bouc-Wen model was used to describe the hysteresis in piezoelectric actuators and parameters were identified to characterize the hysteresis using a genetic algorithm. To improve the performance of the system, a model reference adaptive controller was designed based on the Lyapunov stability theory. Both numerical simulations and experimental results imply that the controller was effective in eliminating the hysteresis in the piezoelectric actuators.

Key Words

Micro-operating mechanism, parallel mechanism, piezoelectric actuator, parameter identification, adaptive control

1. Introduction

In biology, micro-machining, and industry, there is a need to manipulate micro-objects in adverse environments, that may be poisonous, corrosive, or simply too remote for direct human contact. Three-dimensional high-speed micro-manipulation is a fundamental technology for micro-mechatronics and bioengineering applications [1]–[5]. Recently, great progress has been made in biology through advances in biotechnology, including genetic engineering, cellular engineering, and developmental engineering. Such research requires micro- and nano-manipulation and mass production as well as repetitive, high-throughput, and high-precision processing. The principal problem is the size of the objects to be manipulated when the object is extremely small, such as a cell or embryo [6]. A complex fine-motion control system is required. Physical phenomena in the micro-world differ from those in the macro-world. A manipulation system must be designed very carefully and new approaches must be developed to address the challenge of high-speed micro-manipulation.

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The piezoelectric actuators have fast response times, high energy density and high resolution; however, they exhibit hysteresis between input voltage and displacement and between displacement and output force. Considerable effort has been focused on modelling and correcting this non-linear behaviour [7]–[17].

To achieve a high-precision micro-operating mechanism, we used Bouc-Wen model to describe the hysteresis and identified the parameters using genetic algorithm (GA). A model reference adaptive controller was designed to eliminate the hysteresis and noise present in the micro-operating system (Fig. 1).

2. Control Model

2.1 Micro-Operating Mechanism

A Stewart platform [18] is used as the operator, as shown in Fig. 1, and consisted of four parts. The first part was the end plate, to which the operating hand was fixed. The second part consisted of six linkages. The third part consisted of six piezoelectric actuators with equal maximum displacement. The fourth part was the base plate, on which the six piezoelectric actuators were fixed.

Figure 1. Micro-operating mechanism.
2.2 Modelling the Micro-Operating Mechanism

The centre of the base-platform was chosen as the origin in our coordinate system. The position of the centre of on the upper platform $Y(t)$ can be described by:

$$Y(t) = \xi i + \Psi j + \zeta k = F\{y_i(t)\} \quad (i = 1–6) \quad (1)$$

where $y_i(t)$ is the output of the six piezoelectric actuators. From this equation, the system output can be mapped to the displacement of the six piezoelectric actuators [19].

With reference to Fig. 2, the governing equation of one piezoelectric actuator can be obtained as follows:

$$m_i\ddot{x}_i(t) + c_i\dot{x}_i(t) + k_i\dot{x}_i(t) + z_i(t) = F_i(t) - f_{in}(t) \quad (i = 1–6) \quad (2)$$

$$y_i(t) = Cx_i(t)$$

where $m_i$ is the mass of the piezoelectric actuator, $x_i(t)$, $\dot{x}_i(t)$ and $\ddot{x}_i(t)$ are the displacement, velocity and acceleration of the piezoelectric actuator, $c_i$ is the viscous damping coefficient, $k_i$ is the stiffness, and $f_{in}(t)$ is the force generated by the load. In the system the load includes two parts: the load of the external object, and the load generated by the mechanism such as the effect of the displacement of other actuators and noise. The hysteresis in the restoring force of the piezoelectric actuator $z(t)$ is given by Bouc-Wen model [20] as:

$$\ddot{z}_i = A_i\dot{z}_i - \beta_i\dot{z}_i|z_i|^n - \gamma_i|\dot{z}_i|z_i|z_i|^{n-1} \quad (i = 1–6) \quad (3)$$

where $A_i$, $\alpha_i$, $\beta_i$, and $n$ are the parameters of the model. One can control the shape of the hysteresis loop by adjusting $A_i$, $\alpha_i$, and $\beta_i$, and the smoothness of the hysteresis loop by adjusting $n$. In this paper, $n = 1$. From the theory proposed by T. S. Low and W. Guo in [21], [22], a hysteresis model can be included as shown in (5) and we can rewrite the equations of the system as given by (4):

$$m_i\ddot{x}_i(t) + c_i\dot{x}_i(t) + k_i\dot{x}_i(t) = k_i[d_{ei}\dot{u}_i(t) - h_i] + f_{in}(t) \quad (4)$$

where $h_i(t)$ is a state variable, $d_{ei}$ is the piezoelectric coefficient, and $u_i$ is the input.

2.3 Parameter Identification

There were six piezoelectric actuators in the micro-operating mechanism. For each actuator, there were seven parameters to be identified:

$$p_i = \{m_i, c_i, k_i, d_i, \alpha_i, \beta_i, \gamma_i\} \quad (i = 1–6)$$

We adopted a GA to identify these parameters of the non-linear system. The GA proposed by Holland is a technique that can be used to find exact or approximate solutions to optimization and search problems. A flow chart describing the GA is shown in Fig. 3.

To determine the displacement of the piezoelectric actuators, as well as the system input, we used the setup as shown in Fig. 4. A sinusoidal signal was generated by a digital to analog (DA) board as the input to the piezoelectric actuator $u_i$. The input signal was in the range 0–8V and with a frequency of 1 Hz. To reduce the response time, the signal was split prior to the amplification needed to drive the piezoelectric actuators, and sent to an analog to digital (AD) board. The power amplifier allowed an
input voltage of 0–120 V to the piezoelectric actuators, which was their rated voltage. Strain gages were used to measure the displacement of the piezoelectric actuators $x_i$. Each piezoelectric actuator had two high-precision strain gages attached to it: one to measure displacement and the other for temperature compensation. The strain signal was relayed to the workstation using an AD board which sampled at 3 kHz.

Using these measurement devices we obtained the input signal $u_i$ and the output displacement $x_i$ ($i = 1–6$). Using a GA with a maximum number of generations of 100, we were able to identify the hysteresis in the system as shown in Fig. 5 and Table 1.

We can define the error between the measured and the calculated displacement as:

$$\varepsilon_j = |x_j - \hat{x}_j| \quad (j = 1–N)$$

where $N = 21,000$ is the number of displacement samples $x_i$ that was used. If the maximum in the measured displacement is $x_{\text{max},j}$ we can determine $\Delta_j$ as:

$$\Delta_j = \frac{\varepsilon_j}{x_{\text{max},j}} \quad (j = 1–N)$$

If the number of the samples where $\Delta_j < 5\%$ is $n$, we define fitness rate $\sigma$ as:

$$\sigma = \frac{n}{N} \times 100\%$$

The parameter $\sigma$ allowed us to determine how close a group of identified parameters is to the measured data. The values of $\sigma$ that were found are shown in the last row of Table 1. Although the piezoelectric actuators are nominally identical, the input voltage-displacement curves indicate that each piezoelectric actuator had a different hysteresis loop.

![Figure 4. Measurement system.](image)

![Figure 5. Time versus displacement (left) and input voltage versus displacement (right). Calculated data are shown by the black dashed lines and measured data are showed by the black solid lines.](image)
3. Model Reference Adaptive Control (MRAC) of the Micro-Operating Mechanism

An MRAC system is developed in this section. The effect of hysteresis in the piezoelectric actuators can be eliminated using MRAC. For each actuator in the micro-operating mechanism, by using the following relationships:

\[ x_{i1}(t) = x_i(t) \]

\[ x_{i2}(t) = \dot{x}_{i1}(t) \]

we can rewrite (4) and (5) as follows:

\[ \dot{x}_i(t) = A_p x_i(t) + g_k b u_i(t) + \delta_i(t) b \]  \hspace{1cm} (9)
\[
y_i(t) = C_i x_i(t) \quad (i = 1-6)
\]

where

\[
x_i(t) = \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \end{bmatrix}, \quad A_{pi} = \begin{bmatrix} 0 & 1 \\ -\frac{k_i}{m_i} & -\frac{c_i}{m_i} \end{bmatrix}, \quad g_i = \frac{k_i d_{xi}}{m_i},
\]

\[C_i = 1, \quad \delta_i(t) = -\left[ \frac{k_i}{m_i} h_i(t) - f_{0i}(t) \right].\]

Here, a stable reference system is given by:

\[
\dot{x}_{mi}(t) = A_{mi} x_{mi}(t) + g_m b r_i(t)
\]

\[
y_{mi}(t) = C_{mi} x_{mi}(t) \quad (i = 1-6)
\]

where \(x_{mi} \in \mathbb{R}^n\) is the state variable of the reference model, \(r_i(t) \in \mathbb{R}\) is the input of the reference model, and \(y_{mi}(t) \in \mathbb{R}\) is the output of the reference model.

We can define the control law as:

\[
u_i(t) = \theta_{zi} x_i(t) + \theta_{r} r_i(t) + \theta_{bi} \quad (i = 1-6)
\]

where \(\theta_{zi} = [\theta_{zi1} \quad \theta_{zi2}]\). We assumed that the piezoelectric actuators used here had same control model, and so the subscript \(i\) is eliminated for the rest of this section.

Substituting (13) into (9) gives:

\[
\dot{x}(t) = (A_p + gb\theta_x)x + gb\theta_r r(t) + b[g\theta_\delta + \delta(t)]
\]

\[
y(t) = Cx(t)
\]

The system output \(y(t)\) will follow the reference model output \(y_m(t)\) when:

\[
A_p + gb\theta_x^* = A_m
\]

\[
g\theta_r^* \equiv g_m
\]

\[
g\theta_\delta^* + \delta(t) \equiv 0
\]

\[
C_m \equiv C = 1
\]

where \(\theta_x^*, \theta_r^*, \text{ and } \theta_\delta^*\) are the true values of the control gain. Because we do not know the parameters exactly, we use an estimate of the control gains. We define the estimate errors vector \(\phi(t)\) as:

\[
\phi(t) = [\phi_x(t) \quad \phi_r(t) \quad \phi_\delta(t)]^T
\]

\[
\phi_x(t) = \theta_x(t) - \theta_x^*
\]

\[
\phi_r(t) = \theta_r(t) - \theta_r^*
\]

\[
\phi_\delta(t) = \theta_\delta(t) - \theta_\delta^*
\]

and define a state trace error vector as:

\[
\epsilon(t) = x(t) - x_m(t)
\]

Therefore, the dynamics of the state vector are given by:

\[
\dot{\epsilon}(t) = \dot{x}(t) - \dot{x}_m(t) = (A_p + gb\theta_x)x + gb\phi_r r(t) + b[g\phi_\delta + \delta(t)] - A_m x_m(t) - g_m b r(t)
\]

We then substitute the estimates of the error and the state error into (18) to obtain:

\[
\dot{\epsilon}(t) = A_p \epsilon(t) + gb\phi_x x(t) + gb\phi_r r(t) + gb\phi_\delta(t)
\]

\[
= A_p \epsilon(t) + gb\phi(t)^T \omega(t)
\]

where \(\phi(t) = [\phi_x(t) \quad \phi_r(t) \quad \phi_\delta(t)]^T, \omega(t) = [x(t) \quad r(t) \quad 1]^T\).

To make the state trace error converge to zero, the parameters of the adaptive law are given by:

\[
\dot{\theta}(t) = \begin{bmatrix} \dot{\theta}_x(t) \\ \dot{\theta}_r(t) \\ \dot{\theta}_\delta(t) \end{bmatrix} = -\text{sgn}(g) \Gamma \omega(t)e^T(t)Pb
\]

where \(\Gamma\) is the adaptive gain matrix, which is a positive real and diagonal, and \(P\) is a positive symmetric matrix. \(P\) is
calculated using Kalman–Yakubovich lemma. Consider
the reference system governed by (11) and (12). The
transfer function \( G_m(s) = C_m(sI - A_m)^{-1}g_m b \) is strictly
positive real if and only if there exists positive definite
matrices \( P \) and \( Q \) such that:

\[
A_m^T P + PA_m = -Q
\]  

(21)

The reference system is governed by (11) and (12) is
designed as an asymptotically stable and completely control-
able system, and \( A_m \) is a Hurwitz matrix. It follows that
for any positive definite matrix \( Q \), we can determine \( P \).

Theorem 1. Consider the system governed by (4) and
(5). The reference system is described by (11) and (12),
the control law is described by (13) and the adaptive
law is described by (20). We can postulate that \( e(t) \),
\( \phi(t) \), and \( u(t) \) are bounded, and:

\[
\lim_{t \to \infty} \|e(t)\| = 0 \quad (\|\lim_{t \to 0} y(t) - y_m(t)\| = 0)
\]

Proof. To prove this, we define a Lyapunov function as:

\[
V(e(t), \phi(t)) = e^T(t)Pe(t) + |g|\phi^T(t)\Gamma^{-1}\phi(t)
\]

(22)

where \( \Gamma \) and \( P \) are positive definite matrices so \( V(e(t), \phi(t)) \)
is positive definite.

\[
\dot{V}(e(t), \phi(t)) = 2e^T(t)Pe(t) + 2|g|\phi^T(t)\Gamma^{-1}\phi(t)
\]

Substituting (19) into (23) gives:

\[
\dot{V}(e(t), \phi(t)) = 2e^T(t)Pe(t) + 2|g|\phi^T(t)\Gamma^{-1}\phi(t)
\]

\[
= 2e^T(t)P[\dot{A}_m e(t) + gb\phi^T(t)\omega(t)]
\]

\[
+ 2|g|\phi^T(t)\Gamma^{-1}\phi(t)
\]

\[
= e^T(t)(A_m^TP + PA_m)e(t)
\]

\[
+ 2e^T(t)Pg\phi^T(t)\omega(t) + 2|g|\phi^T(t)\Gamma^{-1}\phi(t)
\]

\[
= -e^T(t)Qe(t) + 2e^T(t)Pg\phi^T(t)\omega(t)
\]

\[
+ 2|g|\phi^T(t)\Gamma^{-1}\phi(t)
\]

(23)

where \( \phi(t) = \theta(t) - \theta^*, \phi(t) = \theta(t) \). Substituting the adaptive
law (20) into (24) yields:

\[
\dot{V}(e(t), \phi(t)) = -e^T(t)Qe(t) \leq 0
\]

(25)

Therefore, \( V(e(t), \phi(t)) \) is a bounded function, and \( e(t) \)
and \( \phi(t) \) are bounded.

When \( \phi(t) = \theta(t) - \theta^* \), \( \theta(t) \) is bounded, and so the
control law \( u(t) = \theta_x x(t) + \theta_r r(t) + \theta_b \) is also bounded.

When \( \dot{e}(t) = A_m e(t) + gb\phi^T(t)\omega(t), \) where \( \omega(t) = [x(t) \ r(t) 1]^T \)
is bounded, it can be shown that \( \dot{e}(t) \) is
also bounded.

When \( \dot{e}(t) \) is bounded, we can define a term \( \varepsilon \) such that
\( \varepsilon \in R \) and \( \int_{t}^{\infty} \varepsilon^T(\tau)Qe(\tau)d\tau \leq \varepsilon \), then in the limit
\( \lim_{t \to \infty} \|e(t)\| = 0 \), it follows that is \( \lim_{t \to 0} x(t) - x_m(t) = \lim_{t \to 0} y(t) - y_m(t) = 0 \).

4. Numerical Simulation

4.1 Numerical Simulation for Each Piezoelectric
Actuator

A tracking control numerical simulation for each piezoelec-
tric actuator described in this section. The control model
for the piezoelectric actuators is given by (9) and (10). All
the parameters are shown in Table 1. In the numerical
simulations, \( f_0 \) is described by:

\[
f_0 = (0.1 + \sin(50\pi t) + \sin(100\pi t)) \times (1e - 6)
\]

(26)

The same reference system was used for the six actu-
tors, given by (11) and (12), where \( A_m = \begin{bmatrix} 0 & 1 \\ -1600 & -64 \end{bmatrix} \).

The reference signal \( r_i(t) \) is described by (27) and is shown in Fig. 6:

\[
r_i(t) = \sin(0.01\pi t) + 6\sin(0.2\pi t) + \sin(\pi t) \quad (i = 1-6)
\]

(27)

For each piezoelectric actuator the adaptive gain matrices

\[
\Gamma_i = \begin{bmatrix} \gamma_{ix} & \gamma_{ir} \\ \gamma_{ix} & \gamma_{ir} \end{bmatrix}
\]

are given in Table 2.

The initial states of the simulations are \( x_{i1}(0) = 0, x_{i2}(0) = 0, x_{i3}(0) = 0, \) and \( x_{i4}(0) = 0 \). The results are
shown in Figs. 7 and 8.

Figure 7 shows simulated data for the piezoelectric
transducer displacement with and without the MRAC loop.
From Fig. 7(a), the output of the actuator followed the
output of the reference model well. The circled areas in
Fig. 7(b) show the error between the open-loop output
and the ideal output, which result from the non-linear
restoring force. Although the errors due to the non-linear

![Figure 6. Reference input signal.](image-url)
Table 2
Adaptive Gains.

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<tr>
<td>$\gamma_{i\delta}$</td>
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Figure 7. Simulation results: (a) actuator output with an MRAC controller; (b) actuator output of an open-loop system. The solid black lines are the output of the reference model and the black dashed lines are the output of the piezoelectric actuator with an MRAC controller.

Figure 8. Simulation results: (a) input voltage of the MRAC controller; (b) tracking error between the actuator and reference model.

The restoring force were eliminated using the MRAC, there was some deviation between the input and output in the circle regions in Fig. 8(a). The error in region of Circle 1 shows that actuator output could not follow the reference model at very high speeds. The response can be improved by adjusting the reference model and the adaptive gains. We can determine the error in the region of Circle 2 which is less than 0.25 μm, from Fig. 8(b).
Figure 9. Time versus displacement: (a) reference outputs; (b) piezoelectric actuators outputs with MRAC controller.

Figure 10. Tracking results of the micro-operating system: (a) trace of the top platform centre; (b) time response in the $x$ direction; (c) $y$ direction; and (d) $z$ direction.
4.2 Numerical Simulation for the Micro-Operating System

A tracking control numerical simulation for each piezoelectric actuator described in this section. The output of the micro-operating system is given by (1). In the experiment the centre of the top platform will track a circle described by:

\[
\begin{align*}
    x &= r \cos \theta \\
    y &= r \sin \theta \\
    z &= C
\end{align*}
\]

where \( r = 2 \mu \text{m} \) and \( C = 0.100002 \text{m} \). The adaptive gains are given in Table 2. The reference displacements of the piezoelectric actuators were calculated with the trace described by \( (28) \), and are shown in Fig. 9(a). Figure 9(b) shows the numerical results of the piezoelectric actuators outputs with the MRAC controller. The actuators could not follow the reference displacement quickly. This result is similar to that shown in Fig. 9. We can improve the performance of the actuators by adjusting the adaptive gains and the reference model in the MRAC system.

Figure 10 shows numerical results, for the micro-operating system. The Newton iteration was used to solve its attitude. The precision of the Newton iteration were \( 1 \times 10^8 \text{m} \), although a higher precision will improve the accuracy of the system, it will make the system too slow for real-time applications. These results indicate that the MRAC controller allowed the micro-operating system to follow the reference signal and the non-linear restoring forces were eliminated.
5. Experimental Results

The tracking experiments for the actuators are described in this section. The micro-operating mechanism is shown in Fig. 1, and the control system is shown in Fig. 11. In the experiment the actuators tracked sinusoidal signals, and the reference signal voltage was given by $y_r = 2\sin(40\pi t) + 2$. The adaptive gains in the experiment were controlled based on the data from numerical simulations. The adaptive gains are shown in Table 3. The sampling frequency was set to 100 Hz. The reference model was described by (11) and (12), where $A_m = \begin{bmatrix} 0 & 1 \\ -1600 & -64 \end{bmatrix}$, $g_m = 2.72e^{-3}$, and $C_m = [1 \ 0]$, which were chosen to be the same as the reference model used in the numerical simulations. Experimental data for one actuator are shown in Figs. 12–15.

Figure 12 shows the output of the piezoelectric actuator and the reference model. Figure 13 shows the velocity of the piezoelectric actuator and the reference model.

Figure 14 shows the tracking error which was less than 0.5 μm when the piezoelectric actuator was stable. Figure 15 shows the input of one of the piezoelectric actuators.

From these experimental results, the output of the piezoelectric actuator could follow the reference model both in displacement and velocity, and the hysteresis of the piezoelectric actuator was eliminated using the MRAC system. However, the tracking speed and tracking error have to be improved by adjusting of the adaptive gains and the reference model. Additionally, the measurement technique should be improved to obtain higher precision.

6. Conclusions

A micro-operating mechanism for a human-scale tele-operating system was proposed. A model of the micro-operating mechanism was built with six piezoelectric actuators based on the Bouc-Wen model. The parameters of the model were identified using a GA. Experimental results showed that the identified results very close to the true values. An MRAC system was designed to improve the performance of the micro-operating system. The results of both the numerical simulation and the tracking experiment implied that the use of MRAC improved the performance of the micro-operating system and the noise and hysteresis in the piezoelectric actuators were eliminated. In the future, other algorithms will be adopted and the MRAC will be improved to make the micro-operating system get higher precision.

References


**Biographies**

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