Proceeding of the IEEE
International Conference on Robotics and Biomimetics (ROBIO)
Shenzhen, China, December 2013

Nonlinear Path Following for Water-jet-based Spherical Underwater Vehicles

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***Abstract*—A command filtered back-stepping path following control is exploited to design a dynamic state-feedback controller for a Water-jet-based Spherical Underwater Vehicles, which obviates the computation of analytic derivatives in the traditional back-stepping design. The ISS-modular approach provides a simple and effective way for controlling non-linear Underwater Vehicle satisfying the strict-feedback form, simultaneously solving the problem of "explosion of complexity" in conventional back-stepping approach by combining command filtered back-stepping, sliding-mode-based integral filters and Input-to-State Stability (*ISS*) analysis. The stability analysis of the closed-loop system is verified based on the small-gain theorem. Numerical simulations illustrate the performance of the proposed nonlinear control method.**

I. INTRODUCTION

THE various interests of Underwater Vehicles have been focused for a long time due to the ability of fully developing abundant marine resources[1]-[4], such as mineral, halo bios and ocean energy, with the feather of unoccupied, reliable and highly maneuverable. A key issue in making Underwater Vehicles feasible and efficient is the control design, consisting of trajectory tracking and path following. The paper concerns the nonlinearity character upon the Underwater Vehicles, and focuses on the nonlinear controller providing accurate path following.

Recently, kinds of techniques have received intensive identified in Underwater Vehicles control: back-stepping approach [5]-[10], PID control [11]-[12], Slide-Mode Control [13].etc. However, the conventional back-stepping technique suffers from the problem of "explosion of complexity"[14].That is, implementation of back-stepping becomes increasingly complex even hard to deduce the analytical expressions, as the order of the system increases, which is mainly driven by computing virtual control derivatives at each step of back-stepping design. Various schemes have been investigated to solve the problem. The dynamic surface control(DSC) technique[15]is proposed by introducing first-order filter of the synthetic virtual control; A command filtered back-stepping technique is designed, lying in compensated tracking errors that retain the standard stability properties of traditional back-stepping approach[16]-[17].

In the paper, a ISS-modular command filtered back-stepping control, obviating the analytic computation of virtual control derivatives, is exploited for state-feedback path following of a generic Underwater Vehicles falling into the category of strict-feedback form. Different from the aforementioned command filtered back-stepping control, the input-to-state stability(ISS) analysis[18]-[19]and the small gain theorem[20] rather than constructing an overall Lyapunov function is adopted for the closed-loop system, which is composite of interconnected controller subsystem and filter subsystem. The command filtered back-stepping approach achieves "ISS-modularity" of the controller-filter pair, subsequently providing that the small-gain theorem[20]is satisfied to guarantee the stability and performance of closed-loop system.

The organization of the paper is outlined as follows. The equations of motion for underwater vehicle are described in section 2.In section 3, the path following control system adopting the ISS-modular command filtered back-stepping technique, and a small-gain theorem based comprehensive stability for the closed-loop system is addressed. Numerical simulations performed on the underwater vehicle nonlinear model demonstrate the effectiveness of the proposed control scheme in Section 4. Finally, concluding remarks end the paper in Section 5.

II. MODELING OF WATER-JET-BASED SPHERICAL UNDERWATER VEHICLES

This paper mainly lies in the research on an underwater vehicle possessing three features: spherical shape, a multiple vectored water-jet-based propulsion system, and internal installation, to improve the motion flexibility. The conceptual design and prototype of spherical underwater vehicle is presented in Figure 1.

The research is supported by Key Research Program of the Natural Science Foundation of Tianjin(13JCZDJC26200), and Tianjin City High School Science &Technology Fund Planning Projection(20120830).

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978-1-4799-2744-9/13/$31.00 ©2013 IEEE

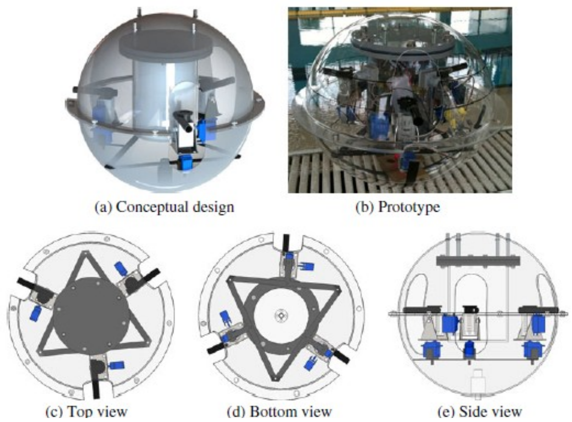


Fig. 1 The overall design of the spherical underwater vehicle

The mathematical model of the spherical underwater vehicle in six degrees of freedom under the assumption of wave-less underwater environment, is described in strict-feedback form:

$$\begin{aligned} &\dot{\eta} = J(\eta)\upsilon \\ &M\dot{\upsilon} + \mathrm{C}(\upsilon)\upsilon + \mathrm{D}(\upsilon)\upsilon + \mathrm{G}(\eta) = \tau \end{aligned} \tag{1}$$

Where vectors $\eta = [x, y, z, \phi, \theta, \psi]^T$ denote the position and attitude, $\upsilon = [u, v, w, p, q, r]^T$ denote thelinear velocity and angular velocity of spherical underwater vehicle, respectively. The transformation matrix $J(\eta)$ is defined as:

$$J(\eta) = diag\{J_1(\eta_2), J_2(\eta_2)\} \tag{2}$$

The expressions of $J_1(\eta_2)$ and $J_2(\eta_2)$ are involved in the Appendix. he vector $\tau \in \Re^6$ is control inputs vector $\tau := \begin{bmatrix} \tau_u & \tau_v & \tau_w & \tau_p & \tau_q & \tau_r \end{bmatrix}^T$. The matrix $M \in \Re^{6\times6}$ is inertia mass including added mass, i.e. $M = M_{RB} + M_A$,

$$M_{RB} = \begin{bmatrix} mI_{3\times3} & -mS(r_G) \\ mS(r_G) & I_0 \end{bmatrix} = diag\{m, m, m, I_x, I_y, I_z\}$$

where m is rigid body mass, $I_{3\times3}$ is an identity matrix and I_0 is inertia tensor matrix. The added mass matrix can be expressed as:

$$M_A = \begin{bmatrix} M_{A11} & M_{A12} \\ M_{A21} & M_{A22} \end{bmatrix} = -diag\{X_{\dot{u}}, Y_{\dot{v}}, Z_{\dot{w}}, K_{\dot{p}}, M_{\dot{q}}, N_{\dot{r}}\}$$

where M_{A11} is added mass, M_{A22} is added inertia tensor, M_{A12} and M_{A21} are added static moments. Hence, inertia mass matrix is deduced as:

$$\begin{aligned} M &= diag\{m - X_{\dot{u}}, m - Y_{\dot{v}}, m - Z_{\dot{w}}, I_x - K_{\dot{p}}, I_y - M_{\dot{q}}, I_z - N_{\dot{r}}\} \\ &:= diag\{m_{11}, m_{22}, m_{33}, m_{44}, m_{55}, m_{66}\} \end{aligned} \tag{3}$$

The matrix $\mathrm{C}(\upsilon) \in \Re^{6\times6}$ is coriolis and centripetal matrix which satisfies $\mathrm{C}(\upsilon) = \mathrm{C}_{RB} + \mathrm{C}_A$. The rigid body coriolis and centripetal matrix C_{RB} is:

$$\mathrm{C}_{RB} = \begin{bmatrix} 0_{3\times3} & -mS(\upsilon_1) - mS(S(\upsilon_2)r_G) \\ -mS(\upsilon_1) - mS(S(\upsilon_2)r_G) & -S(I_0\upsilon_2) + mS(S(\upsilon_1)r_G) \end{bmatrix}$$

$$= \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 0 & mw & -mv \\ 0 & 0 & 0 & -mw & 0 & mu \\ 0 & 0 & 0 & mv & -mu & 0 \\ \hline 0 & mw & -mv & 0 & I_z r & -I_y q \\ -mw & 0 & mu & -I_z r & 0 & I_x p \\ mv & -mu & 0 & I_y q & -I_x p & 0 \end{array}\right]$$

Added mass coriolis and centripetal matrix C_A is:

$$\mathrm{C}_A = \begin{bmatrix} 0_{3\times3} & -S(M_{A11}\upsilon_1 + M_{A12}\upsilon_2) \\ -S(M_{A11}\upsilon_1 + M_{A12}\upsilon_2) & -S(M_{A21}\upsilon_1 + M_{A22}\upsilon_2) \end{bmatrix}$$

$$= \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v \\ 0 & 0 & 0 & Z_{\dot{w}}w & 0 & -X_{\dot{u}}u \\ 0 & 0 & 0 & -Y_{\dot{v}}v & X_{\dot{u}}u & 0 \\ \hline 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v & 0 & -N_{\dot{r}}r & M_{\dot{q}}q \\ Z_{\dot{w}}w & 0 & -X_{\dot{u}}u & N_{\dot{r}}r & 0 & -K_{\dot{p}}p \\ -Y_{\dot{v}}v & X_{\dot{u}}u & 0 & -M_{\dot{q}}q & K_{\dot{p}}p & 0 \end{array}\right]$$

Then the coriolis and centripetal matrix $\mathrm{C}(\upsilon)$ is represents by:

$$\begin{aligned} \mathrm{C}(\upsilon) &= \mathrm{C}_{RB} + \mathrm{C}_A \\ &= \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 0 & m_{33}w & -m_{22}v \\ 0 & 0 & 0 & -m_{33}w & 0 & m_{11}u \\ 0 & 0 & 0 & m_{22}v & -m_{11}u & 0 \\ \hline 0 & m_{33}w & -m_{22}v & 0 & m_{66}r & -m_{55}q \\ -m_{33}w & 0 & m_{11}u & -m_{66}r & 0 & m_{44}p \\ m_{22}v & -m_{11}u & 0 & m_{55}q & -m_{44}p & 0 \end{array}\right] \end{aligned} \tag{4}$$

The damping matrix $\mathrm{D}(\upsilon) = \mathrm{D}_l(\upsilon) + \mathrm{D}_q(\upsilon) \in \Re^{6\times6}$ includes linear damping force $\mathrm{D}_l(\upsilon)$:

$$\mathrm{D}_l(\upsilon) = diag\{d_{l11}, d_{l22}, d_{l33}, d_{l44}, d_{l55}, d_{l66}\}$$

which is related to viscosity of the fluid, and quadratic damping force $\mathrm{D}_q(\upsilon)$ related to the vehicle shape:

$$\mathrm{D}_q(\upsilon) = diag\{d_{q11}|u|, d_{q22}|v|, d_{q33}|w|, d_{q44}|p|, d_{q55}|q|, d_{q66}|r|\}$$

The restoring forces and moments vector is denote as: $\mathrm{G}(\eta) = [\mathrm{G}_u(\eta), \mathrm{G}_v(\eta), \mathrm{G}_w(\eta), \mathrm{G}_p(\eta), \mathrm{G}_q(\eta), \mathrm{G}_r(\eta)] \in \Re^6$.

III. Nonlinear path following Control

The equations of motion for spherical underwater vehicle are decomposed into two functional subsystems, namely, position/attitude subsystem and Linear/angular velocity subsystem. The position/attitude subsystem is controlled by virtual input; while the Linear/angular velocity subsystem is controlled by the available inputs control. Then an ISS modular command filtered back-stepping approach is investigated to provide stable tracking of the position/attitude reference trajectories. Herein the analytic deduction of virtual control derivatives in traditional back-stepping method is avoid and estimated by sliding-mode-based integral filter, consequently the problem of "explosion of complexity" is solved in the proposed method.

The proposed modified back-stepping control approach consists of the interconnected controller-filter pair, which both satisfies the *input-to-state stability(ISS)* condition. Namely, the controller achieves *ISS* with respect to the filter errors meanwhile the filter achieves *ISS* with respect to the system state tracking errors, thence the *small-gain theorem* is employed to ensure the stability of the closed-loop system. The main work of ***III.A*** lies in the present of *ISS*-modular command filtered back-stepping controller. A sliding-mode-based integral filter is advanced and the stability of the overall closed-loop system is discussed in ***III.B***.

A. Command filtered back-stepping controller design

In the section, the command filtered back-stepping approach is investigated bases on the coordinate transformation: $\tilde{\eta}=\eta-\eta_{ref}, \tilde{\upsilon}=\upsilon-u_\eta$, with η_{ref} denotes the position/attitude reference trajectories and u_η being the stabilizing virtual control law in position/attitude subsystem. In position/attitude subsystem, a desired feedback control u_η^0 is firstly designed and is approximated by a sliding-mode-based integral filter raised later. Then, a stabilizing virtual control function u_η is deduced; the true control law $\tau=u_\upsilon$ is defined similarly in linear/angular velocity subsystem.

Step 1:

The position/attitude tracking error dynamic is:

$$\dot{\tilde{\eta}}=\dot{\eta}-\dot{\eta}_{ref}=J(\eta)\upsilon-\dot{\eta}_{ref} \tag{5}$$

A smooth feedback control u_η^0 is defined as:

$$u_\eta^0=J(\eta)^{-1}\dot{\eta}_{ref} \tag{6}$$

Yielding that:

$$\dot{\tilde{\eta}}=J(\eta)(\upsilon-u_\eta^0+u_\eta^0)-\dot{\eta}_{ref}=J(\eta)(\upsilon-u_\eta^0) \tag{7}$$

Let

$$u_\eta=-J(\eta)^{-1}k_\eta\tilde{\eta}+u_\eta^f \tag{8}$$

where $k_\eta\in\Re^{6\times6}$ is a positive matrix specified later, and u_η^f is employed to approximate u_η^0.

Then the dynamic of position/attitude tracking error is governed by:

$$\dot{\tilde{\eta}}=J(\eta)(\tilde{\upsilon}+u_\eta-u_\eta^0)=J(\eta)(\tilde{\upsilon}-u_\eta^0+u_\eta^f)-k_\eta\tilde{\eta} \tag{9}$$

Step 2:

The derivative of linear/angular velocity tracking error $\tilde{\upsilon}$ is:

$$\dot{\tilde{\upsilon}}=\dot{\upsilon}-\dot{u}_\eta=M^{-1}\left(\tau-\mathrm{C}(\upsilon)\upsilon-\mathrm{D}(\upsilon)\upsilon-\mathrm{G}(\eta)\right)-\dot{u}_\eta \tag{10}$$

The smooth feedback control is:

$$\begin{aligned}u_\upsilon^0=&-M(-M^{-1}\mathrm{C}(\upsilon)\upsilon-M^{-1}\mathrm{D}(\upsilon)\upsilon-M^{-1}\mathrm{G}(\eta)-\dot{u}_\eta\\&+J^T(\eta)\tilde{\eta})\end{aligned} \tag{11}$$

Yielding that:

$$\begin{aligned}\dot{\tilde{\upsilon}}&=M^{-1}\left(\tau-u_\upsilon^0+u_\upsilon^0-\mathrm{C}(\upsilon)\upsilon-\mathrm{D}(\upsilon)\upsilon-\mathrm{G}(\eta)\right)-\dot{u}_\eta\\&=M^{-1}\left(\tau-u_\upsilon^0\right)-J^T(\eta)\tilde{\eta}\end{aligned} \tag{12}$$

Let

$$\tau=-Mk_\upsilon\tilde{\upsilon}+u_\upsilon^f \tag{13}$$

where $k_\upsilon\in\Re^{6\times6}$ is a positive matrix specified later, and u_υ^f is employed to approximate the feedback control u_υ^0.

The linear/angular velocity tracking error $\tilde{\upsilon}$ is governed by:

$$\begin{aligned}\dot{\tilde{\upsilon}}&=M^{-1}\left(-Mk_\upsilon\tilde{\upsilon}+u_\upsilon^f-u_\upsilon^0\right)-J^T(\eta)\tilde{\eta}\\&=-k_\upsilon\tilde{\upsilon}+M^{-1}\left(u_\upsilon^f-u_\upsilon^0\right)-J^T(\eta)\tilde{\eta}\end{aligned} \tag{14}$$

Considering the Eqs.(9) and (14), the position/attitude and linear/angular velocity tracking error subsystem is denoted as:

$$\begin{aligned}\dot{\tilde{\eta}}&=J(\eta)(\tilde{\upsilon}-u_\eta^0+u_\eta^f)-k_\eta\tilde{\eta}\\\dot{\tilde{\upsilon}}&=-k_\upsilon\tilde{\upsilon}+M^{-1}\left(u_\upsilon^f-u_\upsilon^0\right)-J^T(\eta)\tilde{\eta}\end{aligned} \tag{15}$$

The state error subsystem (15) can be proved to be *ISS* with respect to the filter error subsystem $\tilde{u}$ in ***Lemma 1***.

Lemma 1. The state error subsystem (15), viewed as a system with states $\tilde{x}=[\tilde{\eta}^T,\tilde{\upsilon}^T]^T$, and inputs $\tilde{u}=u^f-u^0$ $=[(u_\eta^f-u_\eta^0)^T,(u_\upsilon^f-u_\upsilon^0)^T]^T:=[\tilde{u}_\eta^T,\tilde{u}_\upsilon^T]^T$, satisfies the ***input-to-state stability***.

Proof. Consider the *ISS*-Lyapunov function candidate $V_{\tilde{x}}=\frac{1}{2}\tilde{x}^T\tilde{x}=\frac{1}{2}(\tilde{\eta}^T\tilde{\eta}+\tilde{\upsilon}^T\tilde{\upsilon})$. Its derivative along the trajectories (15) is:

$$\begin{aligned}\dot{V}_{\tilde{x}}&=\tilde{\eta}^T\dot{\tilde{\eta}}+\tilde{\upsilon}^T\dot{\tilde{\upsilon}}\\&=\tilde{\eta}^T\left[J(\eta)(\tilde{\upsilon}-u_\eta^0+u_\eta^f)-k_\eta\tilde{\eta}\right]\\&\quad+\tilde{\upsilon}^T\left[-k_\upsilon\tilde{\upsilon}+M^{-1}\left(u_\upsilon^f-u_\upsilon^0\right)-J^T(\eta)\tilde{\eta}\right]\\&=-\tilde{\eta}^Tk_\eta\tilde{\eta}+\tilde{\eta}^TJ(\eta)\tilde{\upsilon}+\tilde{\eta}^TJ(\eta)\tilde{u}_\eta\\&\quad-\tilde{\upsilon}^Tk_\upsilon\tilde{\upsilon}+\tilde{\upsilon}^TM^{-1}\tilde{u}_\upsilon-\tilde{\upsilon}^TJ^T(\eta)\tilde{\eta}\\&=-\tilde{\eta}^Tk_\eta\tilde{\eta}+\tilde{\eta}^TJ(\eta)\tilde{u}_\eta-\tilde{\upsilon}^Tk_\upsilon\tilde{\upsilon}+\tilde{\upsilon}^TM^{-1}\tilde{u}_\upsilon\end{aligned} \tag{16}$$

By completion of squares, the following inequalities hold:

$$\begin{aligned}-\tfrac{1}{2}\tilde{\eta}^Tk_\eta\tilde{\eta}+\tilde{\eta}^TJ(\eta)\tilde{u}_\eta&\le-\tfrac{1}{2}\lambda_{\min}(k_\eta)\tilde{\eta}^T\tilde{\eta}+\|J\|\|\tilde{\eta}\|\|\tilde{u}_\eta\|\\&\le\tfrac{1}{2}\|J\|^2\|\tilde{u}_\eta\|^2\big/\lambda_{\min}(k_\eta);\\-\tfrac{1}{2}\tilde{\upsilon}^Tk_\upsilon\tilde{\upsilon}+\tilde{\upsilon}^TM^{-1}\tilde{u}_\upsilon&\le-\tfrac{1}{2}\lambda_{\min}(k_\upsilon)\tilde{\upsilon}^T\tilde{\upsilon}+\|M^{-1}\|\|\tilde{\upsilon}\|\|\tilde{u}_\upsilon\|\\&\le\tfrac{1}{2}\|M^{-1}\|^2\|\tilde{u}_\upsilon\|^2\big/\lambda_{\min}(k_\upsilon)\end{aligned}$$

Herein $\lambda_{\min}(\bullet)$ denote the minimum eigenvalue of a square matrix .Then the derivative of $V_{\tilde{x}}$ satisfies:

$$\begin{aligned}\dot{V}_{\tilde{x}}\le&-\tfrac{1}{2}\tilde{\eta}^Tk_\eta\tilde{\eta}+\tfrac{1}{2}\|J\|^2\|\tilde{u}_\eta\|^2\big/\lambda_{\min}(k_\eta)\\&-\tfrac{1}{2}\tilde{\upsilon}^Tk_\upsilon\tilde{\upsilon}+\tfrac{1}{2}\|M^{-1}\|^2\|\tilde{u}_\upsilon\|^2\big/\lambda_{\min}(k_\upsilon)\end{aligned} \tag{17}$$

For simplicity, denote $k_{\Theta\tilde{u}},k_{\omega\tilde{u}}$ as:

$k_{\eta\tilde{u}} := \frac{1}{2}\|J\|^2 / \lambda_{\min}(k_\eta), k_{\upsilon\tilde{u}} := \frac{1}{2}\|M^{-1}\|^2 / \lambda_{\min}(k_\upsilon)$.

Yielding that:

$$\begin{aligned}\dot{V}_{\tilde{x}} &\le -\tfrac{1}{2}\tilde{\eta}^T k_\eta \tilde{\eta} + k_{\eta\tilde{u}}\|\tilde{u}_\eta\|^2 - \tfrac{1}{2}\tilde{\upsilon}^T k_\upsilon \tilde{\upsilon} + k_{\upsilon\tilde{u}}\|\tilde{u}_\upsilon\|^2 \\ &\le -\tfrac{1}{4}\tilde{\eta}^T k_\eta \tilde{\eta} - \tfrac{1}{4}\tilde{\upsilon}^T k_\upsilon \tilde{\upsilon} \\ &\le -\tfrac{1}{4}\lambda_{\min}(k_\eta)\|\tilde{\eta}\|^2 - \tfrac{1}{4}\lambda_{\min}(k_\upsilon)\|\tilde{\upsilon}\|^2 \end{aligned} \tag{18}$$

$$\sqrt{4k_{\eta\tilde{u}}/\lambda_{\min}(k_\eta)}\,\|\tilde{u}_\eta\| \le \|\tilde{\eta}\|$$

$$\sqrt{4k_{\upsilon\tilde{u}}/\lambda_{\min}(k_\upsilon)}\,\|\tilde{u}_\upsilon\| \le \|\tilde{\upsilon}\|$$

Since

$$\|\tilde{x}\| \ge \sqrt{k_{x\tilde{u}}}\,\|\tilde{u}\|$$

implies $\dot{V}_{\tilde{x}} \le -\alpha_{\tilde{x}}(\|\tilde{x}\|)$.

With

$$k_{x\tilde{u}} = \min\{\sqrt{4k_{\eta\tilde{u}}/\lambda_{\min}(k_\eta)}, \sqrt{4k_{\upsilon\tilde{u}}/\lambda_{\min}(k_\upsilon)}\}$$

$$\alpha_{\tilde{x}}(s) = \tfrac{1}{4}\min\{\lambda_{\min}(k_\eta), \lambda_{\min}(k_\upsilon)\}s^2$$

Thus, the state error subsystem (15) is ***ISS***, with gain function[21]:

$$\gamma_1^{\tilde{u}}(s) = \sqrt{k_{x\tilde{u}}}\,s \tag{19}$$

□

B. STABILITY ANALYSIS OF THE CLOSED-LOOP SYSTEM

A Lyapunov-based filter is adopted to approximate the feedback control $u_i^0, i = \eta, \upsilon$ and its derivative in position/attitude and Linear/angular velocity dynamics, obviating the complicate analytical computation. Simultaneously the filter error subsystem achieves the *ISS*-modularity with respect to the state error subsystem. Subsequently the small-gain theorem is adopted to guarantee stability and performance of the entire closed-loop system composed with the interconnected control module and filter module. The following ***Theorem1*** states the main result of the paper.

Theorem 1:

Given the equations of motion (1) for Spherical underwater vehicle, the proposed command back-stepping controller (8) and (13), together with the sliding-mode-based integral filter (20), for bounded initial conditions, the closed-loop system signals remain bounded, and the output (position/attitude) tracking errors converge to a small neighborhood of the origin by choosing the controller gains properly.

Proof:

Herein a modified sliding-mode-based integral filter is presented:

$$\dot{u}_i^f = -\sigma_i(u_i^f - u_i^0) - \kappa_i sign(u_i^f - u_i^0) - \varepsilon_i \tilde{x}_i, \quad i = \eta, \upsilon \tag{20}$$

where $\sigma_i, \kappa_i, \varepsilon_i \in \mathbb{R}^{3\times3}, i = \eta, \upsilon$ are positive matrixes. The natural choice for the filter initial condition is $u_\eta^f(0) = u_\eta^0(0), u_\upsilon^f(0) = u_\upsilon^0(0)$. With the definition that $u^f = [(u_\eta^f)^T, (u_\upsilon^f)^T]^T, u^0 = [(u_\eta^0)^T, (u_\upsilon^0)^T]^T$, system(20) can be deduced as:

$$\dot{u}^f = -\sigma(u^f - u^0) - \kappa sign(u^f - u^0) - \varepsilon\tilde{x} \tag{21}$$

where $\sigma = diag\{\sigma_\eta, \sigma_\upsilon\}, \kappa = diag\{\kappa_\eta, \kappa_\upsilon\}$ and $\varepsilon = diag\{\varepsilon_\eta, \varepsilon_\upsilon\}$.

The filter error dynamics $\tilde{u}$ are rewritten as:

$$\dot{\tilde{u}} = \dot{u}^f - \dot{u}^* = -\sigma\tilde{u} - \kappa sign(\tilde{u}) - \varepsilon\tilde{x} - \dot{u}^* \tag{22}$$

For the $\tilde{u}$ -subsystem(22), considering the *ISS*-Lyapunov function candidate:

$$V_{\tilde{u}} = \tfrac{1}{2}\tilde{u}^T\tilde{u} = \tfrac{1}{2}(\tilde{u}_\eta^T\tilde{u}_\eta + \tilde{u}_\upsilon^T\tilde{u}_\upsilon) \tag{23}$$

Its time derivation along the trajectories (22) is:

$$\begin{aligned}\dot{V}_{\tilde{u}} &= \tilde{u}^T\dot{\tilde{u}} = \tilde{u}^T(-\sigma\tilde{u} - \kappa sign(\tilde{u}) - \varepsilon\tilde{x} - \dot{u}^*) \\ &= -\tilde{u}^T\sigma\tilde{u} - \tilde{u}^T\kappa sign(\tilde{u}) - \tilde{u}^T\varepsilon\tilde{x} - \tilde{u}^T\dot{u}^* \\ &\le -\lambda_{\min}(\sigma)\|\tilde{u}\|^2 - \lambda_{\min}(\kappa)\|\tilde{u}\| + \lambda_{\max}(\varepsilon)\|\tilde{u}\|\|\tilde{x}\| \\ &\quad + \|\tilde{u}\|\|\dot{u}^*\| \end{aligned} \tag{24}$$

where $\lambda_{\max}(\bullet)$ means the maximum eigenvalue of a square matrix .As long as $\lambda_{\min}(\kappa) > \|\dot{u}^*\|$, implies:

$$\dot{V}_{\tilde{u}} \le -\lambda_{\min}(\sigma)\|\tilde{u}\|^2 + \lambda_{\max}(\varepsilon)\|\tilde{u}\|\|\tilde{x}\|$$

$$\le -\tfrac{1}{2}\lambda_{\min}(\sigma)\|\tilde{u}\|^2 + \tfrac{1}{2}(\lambda_{\max}(\varepsilon))^2\|\tilde{x}\|^2 / \lambda_{\min}(\sigma)$$

with the inequality

$$-\tfrac{1}{2}\lambda_{\min}(\sigma)\|\tilde{u}\|^2 + \lambda_{\max}(\varepsilon)\|\tilde{u}\|\|\tilde{x}\| \le \tfrac{1}{2}(\lambda_{\max}(\varepsilon))^2\|\tilde{x}\|^2 / \lambda_{\min}(\sigma)$$

hold by completion of squares.

Then the derivative of $V_{\tilde{u}}$ satisfies:

$$\dot{V}_{\tilde{u}} \le -\tfrac{1}{2}\lambda_{\min}(\sigma)\|\tilde{u}\|^2 + \tfrac{1}{2}(\lambda_{\max}(\varepsilon))^2\|\tilde{x}\|^2 / \lambda_{\min}(\sigma)$$

$$\le -\tfrac{1}{4}\lambda_{\min}(\sigma)\|\tilde{u}\|^2$$

for all $\|\tilde{u}\| > (\sqrt{2}\lambda_{\max}(\varepsilon)/\lambda_{\min}(\sigma))\|\tilde{x}\|$.

Therefore, filter error subsystem (22) viewed as a system with states $\tilde{u}$ and input $\tilde{x}$, is proved to meet *input-to-state stability*, with gain function:

$$\gamma_2^{\tilde{x}}(s) = (\sqrt{2}\lambda_{\max}(\varepsilon)/\lambda_{\min}(\sigma))s \tag{25}$$

Recalling*Lemma1*, the *ISS* property has been established for the state error subsystem (15). Thus, there exist class *KL* functions $\beta_{\tilde{x}}, \beta_{\tilde{u}}$, and class K functions $\gamma_1^{\tilde{u}}, \gamma_2^{\tilde{x}}$, such that:

$$\|\tilde{x}(\bullet)\|_\infty \le \max\{\beta_{\tilde{x}}(\|\tilde{x}(0)\|), \gamma_1^{\tilde{u}}(\|\tilde{u}\|_\infty)\} \tag{26}$$

$$\|\tilde{u}(\bullet)\|_\infty \le \max\{\beta_{\tilde{u}}(\|\tilde{u}(0)\|), \gamma_2^{\tilde{x}}(\|\tilde{x}\|_\infty)\} \tag{27}$$

According to the small-gain theorem[20], by checking the following condition:

$$\gamma_2^{\tilde{x}}(\gamma_1^{\tilde{u}}(\|s\|)) < s \tag{28}$$

We have the condition $(\sqrt{2}\lambda_{\max}(\varepsilon)/\lambda_{\min}(\sigma))\sqrt{k_{x\tilde{u}}}\,s < s$, i.e. $\sqrt{2k_{x\tilde{u}}}\lambda_{\max}(\varepsilon) < \lambda_{\min}(\sigma)$.

Then, for bounded initial conditions $\tilde{x}(0)$ and $\tilde{u}(0)$, we get:

$$\|\tilde{x}(\bullet)\|_\infty \le \max\{\beta_{\tilde{x}}(\|\tilde{x}(0)\|), \gamma_1^{\tilde{u}}\beta_{\tilde{u}}(\|\tilde{u}(0)\|\} \tag{29}$$

$$\|\tilde{u}(\bullet)\|_\infty \le \max\{\beta_{\tilde{u}}(\|\tilde{u}(0)\|), \gamma_2^{\tilde{x}}\beta_{\tilde{x}}(\|\tilde{x}(0)\|)\} \tag{30}$$

which means that the closed-loop system is locally input-to-state stable[20].The boundedness of $(\tilde{x},\tilde{u})$ and consequently, the boundedness of $\eta,\upsilon;u_\eta,u_\upsilon$ and the control signal τ can be guarteed. Thus, all the signals in the closed-loop remain bounded.

In particular, the response of $\tilde{x}$ satisfies:

$$\|\tilde{x}(\bullet)\|_\infty \le \max\{\beta_{\tilde{x}}(\|\tilde{x}(0)\|), \gamma_1^{\tilde{u}}\beta_{\tilde{u}}(\|\tilde{u}(0)\|\}$$

Evidently, $\tilde{\eta}=\eta-\eta_{ref}$ converges to a small neighborhood of the origin by choosing the control parameters properly.

IV. Numerical Simulations

The command filtered back-stepping path control law derived in(8) and (14) has been verified in MATLAB/Simulink® environment in the case of a path following. The nonlinear six-degrees-of-freedom dynamic model of Water-jet-based spherical underwater vehicle[12] is considered, whereas mass parameters in spherical underwater vehicle are given as:

$$\begin{aligned} &m_{11}=23.05, m_{22}=23.05, m_{33}=23.05 \\ &m_{44}=0.4271, m_{55}=0.4271, m_{66}=0.5145 \end{aligned} \tag{31}$$

The damping matrix parameters are provided as:

$$\begin{aligned} &d_{l11}=0.02, d_{l22}=0.02, d_{l33}=0.02, \\ &d_{l44}=3.8e-4, d_{l55}=3.8e-4, d_{l66}=3.8e-4 \\ &d_{q11}=0.3, d_{q22}=0.3, d_{q33}=0.3, d_{q44}=0, d_{q55}=0, d_{q66}=0 \end{aligned} \tag{32}$$

The restoring forces and moments vector is selected for:

$$\begin{aligned} &\mathrm{G}_u(\eta)=\mathrm{G}_v(\eta)=\mathrm{G}_w(\eta) \\ &=\mathrm{G}_p(\eta)=\mathrm{G}_q(\eta)=\mathrm{G}_r(\eta)=0 \end{aligned} \tag{33}$$

The reference commands for position and attitudeare provided as:

$$x_{ref}=\begin{cases} 0, t\le 10 \\ t-10, 10<t\le 20 \\ 14+4\sqrt{2}\sin(\frac{\pi}{8}t+\frac{5\pi}{4}), 20<t\le 28 \\ -0.5t+32, 28<t\le 48 \\ 4+4\sqrt{2}\sin(\frac{\pi}{6}\mathrm{t}-\frac{3\pi}{4}), 48<t\le 54 \end{cases}$$

$$y_{ref}=\begin{cases} 0, t\le 10 \\ t-10, 10<t\le 20 \\ 6+4\sqrt{2}\cos(\frac{\pi}{8}t+\frac{5\pi}{4}), 20<t\le 28 \\ -0.5t+16, 28<t\le 48 \\ -4+4\sqrt{2}\cos(\frac{\pi}{6}\mathrm{t}-\frac{3\pi}{4}), 48<t\le 54 \end{cases} \tag{34}$$

$$z_{ref}=\begin{cases} t, t\le 10 \\ 10, 10<t\le 54 \end{cases}$$

$$\phi_{ref}=\theta_{ref}=\psi_{ref}=0$$

The command filtered back-stepping path controller is tuned by a trial-and-error procedure on the nonlinear spherical underwater vehicle model, as increasing controller gains can increases the following rate but may cause a worse transient performance. The filter parameters $\sigma_\eta,\sigma_\upsilon$ are also tuned in a trial-and-error procedure. The influence of filter parameters $\kappa_\eta,\varepsilon_\eta;\kappa_\upsilon,\varepsilon_\upsilon$ on the performance is limited; simply selecting the eigenvalues larger than zero. The final gains in simulation are selected as:

$k_\eta=diag\{10;10;10;10;10;10\}, k_\upsilon=diag\{1;1;1;1;1;1\}, \sigma_\eta=$
$diag\{1;1;0.5;1;3;1\}, \kappa_\eta=\varepsilon_\eta=diag\{0.2;0.1;0.2;0.1;0.1;0.1\},$
$\sigma_\upsilon=diag\{1;1;5;1;2;1\}, \kappa_\upsilon=\varepsilon_\upsilon=diag\{0.1;0.3;0.2;0.1;0.2;0.1\}$

The simulation figs confirm that the proposed control provides stable path following of reference commands and system states convergence to the corresponding trim points quickly. More specifically, the behavior of position, achieves quick following, shown in Figs. 2. The time history of attitude, linear velocity and angular velocity are involved in Fig. 3, Fig.4 and Fig.5 respectively.

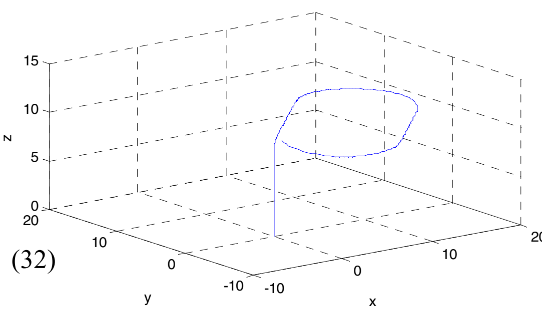
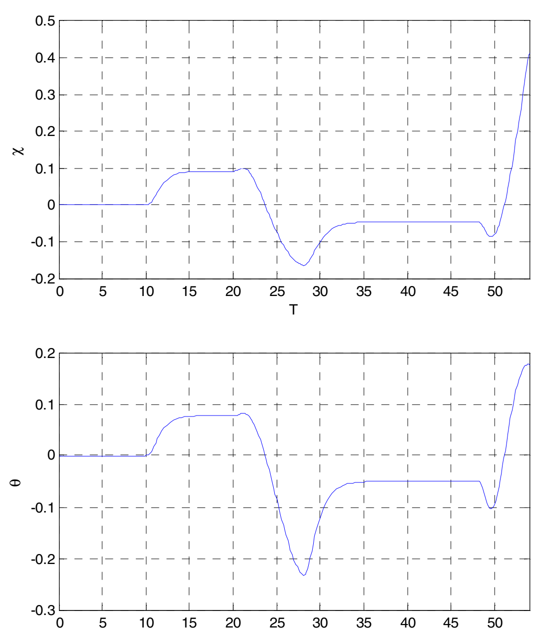


Fig. 2 position response



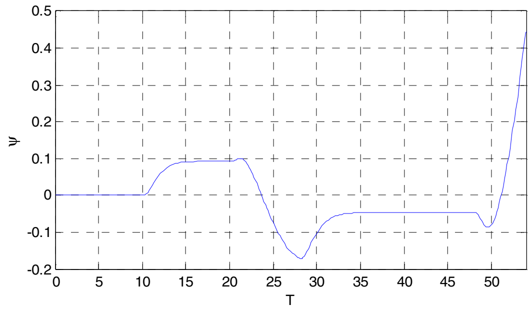


Fig. 3 attitude response

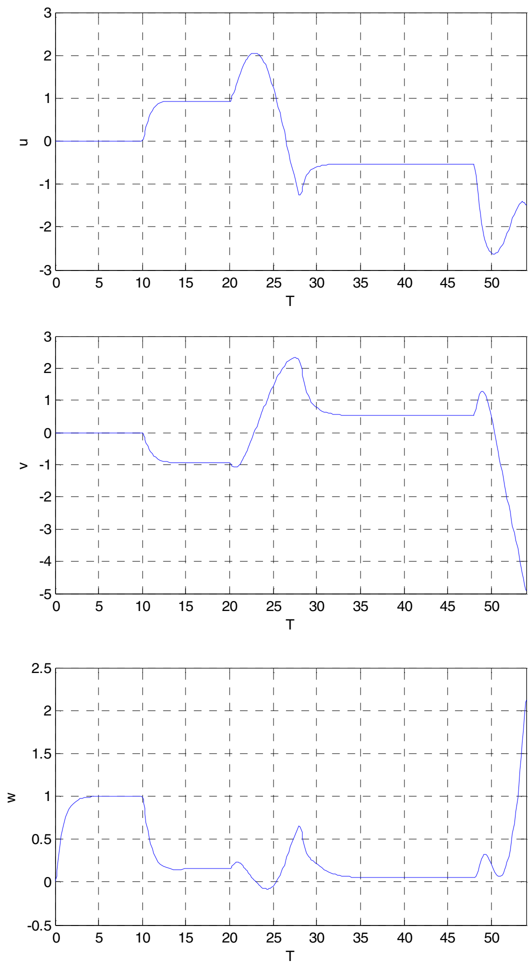
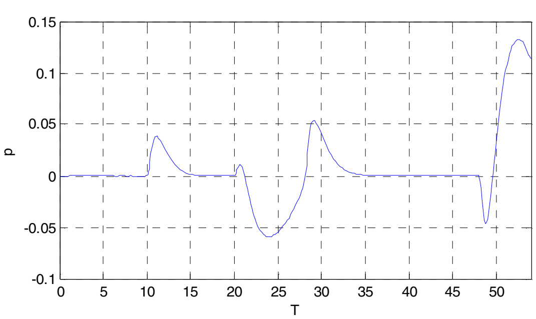


Fig. 4 linear velocity response



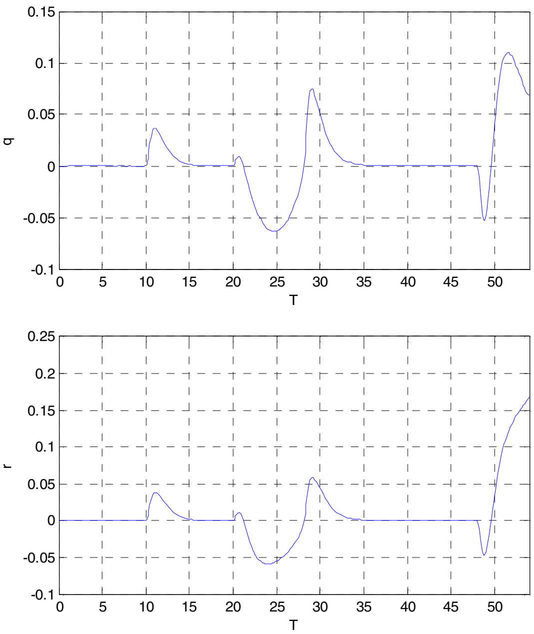


Fig. 5 angular velocity response

The simulation results illustrate that the proposed command filtered back-stepping control scheme for spherical underwater vehicle exhibits relatively excellent path following performance, obviating the complicate analytical derivation of virtual control inputs in traditional back-stepping method, while alleviating simulation computational burden.

V. Conclusion

A nonlinear path following controller based on command filtered back-stepping technology is investigated for the nonlinear model of spherical underwater vehicle. The command filtered back-stepping controller incorporation with *input-to-state stability* analysis is addressed to overcome the "explosion of complexity" problem in conventional back-stepping method. The comprehensive stability analysis is presented utilizing small-gain theorem and guarantees the boundness of closed-loop system. The proposed control scheme has been validated in numerical simulation, exhibiting satisfactory path following performance for the spherical underwater vehicle.

Appendix

The transformation matrixes $J_1(\eta_2), J_2(\eta_2)$ in Eqns. (2)are presented as following:

$$J_1(\eta_2)=\begin{bmatrix}\cos\psi\cos\theta & \cos\psi\sin\theta\sin\phi-\sin\psi\cos\phi & \cos\psi\sin\theta\cos\phi+\sin\psi\sin\phi\\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\phi-\cos\psi\cos\phi & \sin\psi\sin\theta\cos\phi-\cos\psi\sin\phi\\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi\end{bmatrix}$$

$$J_2(\eta_2)=\begin{bmatrix}1 & \sin\phi\tan\theta & \cos\phi\tan\theta\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta\end{bmatrix}$$

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