Catheter Modeling And Tip Force Analysis Based On Mass Spring Model

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Abstract - In the process of vascular intervention surgery, the complex vascular environment in the human body has become the main reason why it is impossible to accurately measure the collision force between the catheter tip and the vascular wall. In this paper, a new measuring method is proposed to solve the problem that the force at the tip of the catheter cannot be measured accurately during the operation. This method uses the mass spring model method to construct the mathematical model of the catheter, and calculates the relationship between the catheter tip force and the comprehensive force based on the law of energy conservation. The experimental results show that this method can calculate the relationship between the catheter tip force and the comprehensive force, and reasonably control the threshold of the comprehensive force through the catheter tip force, which can solve the problem that the sensor needs to be disinfected when it enters the human body and reduce the potential safety hazards of surgery. It is of great significance to the research on the safety of interventional surgery robot in the future.

Index Terms - Vascular interventional surgery, catheter tip, collision force, mass spring model, law of energy conservation

I. INTRODUCTION

Cardiovascular and cerebrovascular diseases endanger people's health all year round. According to China cardiovascular health and disease report 2020, at present, cardiovascular disease death accounts for the first place in the total cause of death of urban and rural residents in China, including 46.66% in rural areas and 43.81% in urban areas. The prevalence of cardiovascular diseases in China is on the rise[1].

Interventional surgery is a new type of high-tech minimally invasive surgery, which has the advantages of less trauma, more indications and less pain. Vascular interventional surgery has a significant effect on the treatment of cardiovascular and cerebrovascular diseases. Interventional surgery refers to pushing or twisting the guide wire catheter in the blood vessel, and then giving drugs to the affected part, so as to dissolve the thrombus and dredge the blood vessels[2].

With the intelligent development of medical devices, vascular interventional robot technology has been widely used in the medical field. Compared with traditional interventional surgery, vascular interventional surgery robot can enable doctors to get rid of the heavy burden caused by wearing heavy lead clothes and reduce the damage caused by radiation absorption. Experiments show that robot assisted PCI can reduce radiation injury by 97%[3]. Moreover, the interventional robot can also break the space constraints brought by traditional surgery, so that doctors and patients thousands of miles apart can realize the operation remotely through network information transmission.

However, due to the high complexity of blood vessels in the human body, once the surgeon is careless, the catheter may damage the blood vessels of human tissues and lead to the failure of the operation. Therefore, the measurement of catheter tip force of vascular interventional surgery robot has become the main research direction in recent years. At present, there are many schemes to measure the catheter tip force of interventional surgery robot. In 2013, Jian Guo team of Tianjin University of technology proposed to place pressure tactile sensors made of pressure-sensitive rubber at the front and side ends of the surgical catheter respectively to directly measure the tip force with human blood vessels, and analyzed the size of the contact force under the premise of different contact areas[4]. In addition, based on the robot catheter operating system with master-slave structure, a new pressure-sensitive rubber tactile sensor array is designed and proposed, which can be installed on the catheter side wall to detect the contact force with the blood vessel wall. With the help of LED lamp array and PC screen, the operator can distinguish the contact position between the tactile sensor array and the blood vessel wall and the force feedback[5]. In 2020, in order to solve the sterilization problem caused by traditional sensors in human blood vessels, Professor Guo's team proposed a method to use virtual sensors to collect the images of the catheter in the blood vessels, extract the shape variables to establish the catheter kinematics model, import the extracted shape variables into the catheter dynamics model, and finally output the collision force between the catheter tip and the blood.
vessel wall[6]. In 2019, the team of Professor Shuxiang Guo from Beijing University of Technology proposed a new detection way to detect the tip force of the catheter, and experiments show that this method is feasible[7]. In 2022, Shuxiang Guo team of Kagawa University in Japan developed a force sensor based on pressure-sensitive rubber, which is used to install on the catheter tip. The measured contact force can be approximately equivalent to the piezoresistance of pressure-sensitive rubber, then measure the collision force between the catheter tip and the blood vessel wall[8].

In the process of vascular interventional surgery, the ability to accurately obtain the force at the tip of the catheter can ensure the safety of the operation. The innovation of this paper is to establish the mass spring model of the whole catheter, and deduce the tip force at the tip of the catheter by using the law of conservation of energy, so as to ensure the safe operation.

This paper is divided into five parts. The first part introduces the interventional robot and its method to obtain the force of catheter tip; The second part introduces the theoretical basis of constructing catheter model; The third part constructs the catheter model according to the law of energy conservation; In the fourth part, the force at the tip of the catheter will be tested and analyzed; The fifth part will summarize this paper.

II. THE OVERVIEW OF SYSTEM PLATFORM

In this paper, the vascular interventional surgery robot system in our laboratory is used for the experiment[9]. This system consists of two parts: master operator and slave operator. The main end operator uses the principle of electromagnetic induction to realize the force feedback function by using double energized coils and magnetic rods, and the slave end operator uses multiple units with clamps to realize the cooperative operation performance of catheter guide wire. When the operator operates the master operator, the operation information of the master is transmitted to the slave operator through the communication serial port. After receiving the signal from the STM32 microcontroller, the slave operator drives the stepping motor to push or rotate the catheter. Therefore, for a series of actions of the master operator, the slave operator can reproduce them in real time and accurately, with good transparency. The overall operation concept diagram is shown in Figure 1. The physical drawing of the main end operator is shown in Figure 2. As shown in Figure 3.

III. THE INTERVENTIONAL CATHETER MODELING

Vascular interventional surgery has been widely used in clinical treatment, but due to the complexity of blood vessels, the acquisition of catheter tip force has become an important part to ensure the safety of surgery. Therefore, it is of great significance to establish the mathematical model of the catheter and find the detection method that can effectively obtain the tip force. In this part, the catheter modeling method of interventional surgery is studied.

Because the catheter is made of non rigid material, the method of elastic modeling is adopted to establish the catheter model. At present, there are many methods to study and solve the modeling of elasticity. The two most common methods are finite element model method and mass spring model method.

Finite element model is a modeling method that discretizes the physical entity by meshing[10]. Its central idea is to discretize the simulation object and treat the real object as the composition of a limited number of simple discrete elements. The physical characteristics of the real object are obtained by analyzing the discrete elements, and then obtain the approximate answer within the accuracy requirements, so as to replace the analysis of the real object[11]. Because most biological soft tissue materials have mechanical properties such as anisotropy, hyperelasticity and viscoelasticity, the finite element method is often used for soft tissue modeling[12]. By analyzing the characteristics of soft tissue and appropriately adjusting the solution equation, a more accurate soft tissue model can be obtained[13]. However, the solution process of the finite element method is very complex. With the gradual refinement of the constitutive equation and discrete element, The more difficult the solving process is to meet the requirements of real-time[14].

The mass spring model is a classical method for solving elastic problems. Its principle is to disperse the mass of the object to be modeled into several uniformly distributed mass
points, which are connected by massless springs following Hooke's law, and then establish a set of equilibrium equations according to the corresponding dynamic equations, continuously solve and update the vertex positions in the equations, and update each point one by one. Its advantages are: the system equation follows the dynamic principle, can truly fit the objective motion law of the object, the model is simple, easy to implement, simple calculation, and can well meet the real-time requirements in the operation process[15]. Through the comparison of the above modeling methods, this paper uses the mass spring method to construct the guide wire catheter model. In the process of pushing the catheter in human blood vessels, there are many forces In view of this analysis, this paper adopts the law of conservation of energy to model and analyze the mass spring of the catheter, and solve the tip force of its tip against the blood vessel.

The catheter modeling mentioned in this section is divided into four sections. In the first section, according to the kinematic model of the catheter bending in the blood vessel, it is discretized into a mass spring model; The second section analyzes the stress of the catheter in the blood vessel; The third section is to establish a mathematical model according to the energy loss of catheter movement. The last section is to obtain the tip force at the end of the catheter according to the comprehensive force of the end clamp and the mathematical model. The specific catheter modeling flow chart is shown in Figure 4.

![Fig. 4 Flow chart of catheter modeling](image1)

**A. Catheter discrete modeling**

Due to the complexity of human blood vessels, the shape of catheter becomes tortuous in interventional surgery. The catheter model is shown in Figure 5. If there is no restriction of human blood vessels on the catheter, the natural state of the catheter is a straight line segment. Therefore, the catheter accumulates elastic potential energy. Therefore, the catheter is discretized into a model connected by a series of particles and springs, as shown in Figure 6. Because the elastic potential energy of the spring depends on the initial and final shape variables of the spring and has nothing to do with the process. When the compression of the spring is the same, the accumulated elastic potential energy is equal[16]. The equivalent model is shown in Figure 7. The above curve AB is regarded as the actual motion trajectory of the catheter in the human blood vessels. Point A represents the puncture point of human femoral artery, and point B represents a certain position of catheter tip in human blood vessel. The following line AB is also connected by a series of particles and springs. At this time, there is compression deformation between the particle and the spring, which can be equivalent to the curve in the figure.

**B. Stress analysis of catheter**

During interventional surgery, the stress of the catheter in the process of being pushed from the end clamp includes the blood flow resistance of the blood to the catheter, the friction between the catheter surface and human blood vessels and other forces.

In real life, each liquid has different degrees of viscosity, and blood is looked upon an incompressible homogeneous Newtonian fluid. During the movement of the catheter, the influence of blood flow resistance needs to be overcome. The effect of blood flow resistance increases gradually with the increase of the length of the catheter pushed into the blood vessel. Because the blood vessel is divided into several circular tubes, the blood flow resistance of the catheter during movement can be equivalent to the sum of the blood flow resistance of each blood vessel segment.

When the blood vessel is cut into very short, the length of the catheter in each section of blood vessel is also short enough to approximate the catheter in each section of blood vessel as a sphere. It is known that the resistance of the sphere can be described by Stokes formula[17]. In 1851, British mathematician and physicist Stokes discovered a formula that can describe this resistance. As shown in Formula (1)[18].

$$F = 6\pi \eta rv$$  \hspace{1cm} (1)

Among them, \(\eta\) is the blood viscosity coefficient, \(r\) is the catheter radius, and \(v\) is the movement speed of the blood flow velocity relative to the catheter, that is, the axial linear speed of the catheter movement is obtained by driving the axial movement motor speed from the end.

In the process of interventional surgery, there is friction between the catheter and the blood vessel wall at all times. According to the data, the friction coefficient of the catheter surface is 0.18[19]. The friction force is shown in formula (2).

$$f = \mu \cdot F_N$$  \hspace{1cm} (2)

Which \(F_N\) represents the pressure on the blood vessel wall after the catheter is inserted into the blood vessel, that is, the gravity of the catheter entering the blood vessel.
C. Analyze the energy loss of the catheter and build model

According to the law of conservation of energy, formula (3) is obtained.

\[ W_1(t) = W_2(t) + W_3(t) + W_4(t) + W_5(t) \]  

Among them, \( W_1(t) \) is the work done by the comprehensive force of the end clamp, \( W_2(t) \) is the work done by the force at the tip of the catheter, \( W_3(t) \) is the elastic potential energy accumulated by the catheter, \( W_4(t) \) is the work done by the blood on the catheter, and \( W_5(t) \) is the work done by the friction between the catheter and the blood vessel wall.

The work done by the comprehensive force measured from the end fixture is shown in formula (4).

\[ W_1(t) = \sum_{i=1}^{n} F_i(t) \cdot \Delta X_i(t) \]  

Where \( F_i(t) \) is the force pushed by each particle to the next particle, \( \Delta X_i(t) \) is the forward distance of each catheter segment, and \( n \) represents that the catheter is divided into \( n \) segments.

The work done by the force at the tip of the catheter is shown in formula (5).

\[ W_2(t) = \sum_{i=1}^{n} f_i(t) \cdot \Delta x_i(t) \]  

Where \( f_i(t) \) is the force pushed by each particle to the next particle after being equivalent to the spring, and \( \Delta x_i(t) \) is the effective distance pushed forward by each catheter segment.

The elastic potential energy accumulated by the catheter is shown in formula (6)[20].

\[ W_3(t) = \sum_{i=1}^{n} \frac{\Delta X_i(t)}{X(t)} k(t) \Delta x(t) \]  

\[ = \sum_{i=1}^{n} \frac{1}{2} k(t) \left( \Delta X_i(t) - \Delta x_i(t) \right)^2 \]  

\[ k(t) = \frac{E \cdot S}{X(t)} \]  

In formulas (6) and (7), \( k(t) \) is the elastic coefficient of the equivalent spring of the catheter, \( E \) is the elastic modulus of the catheter, which is only related to the material of the catheter, \( S \) is the cross-sectional area of the catheter, \( X(t) \) is the actual distance length of the catheter advancing in the blood vessel, and \( x(t) \) is the effective distance length of the catheter advancing in the blood vessel.

The work done by blood to the catheter is shown in formula (8).

\[ W_4(t) = \sum_{i=1}^{n} \Delta X_i(t) \cdot F = \sum_{i=1}^{n} \Delta X_i(t) \cdot 6 \pi \eta \cdot \mu \cdot \Delta x_i(t) \]  

\[ = \sum_{i=1}^{n} 6 \pi \eta \cdot \Delta x_i(t) \]  

\( W_4(t) \) is the work of blood flow resistance on the catheter, \( \eta \) is the blood viscosity coefficient, \( \Delta X_i(t) \) is the actual distance and length of each catheter segment, \( v(t) \) is the relative moving speed of blood and catheter.

The work done by the friction between the catheter and the blood vessel wall is shown in formula (9).

\[ W_5(t) = \sum_{i=1}^{n} \Delta X_i(t) \cdot f_i(t) = \sum_{i=1}^{n} \Delta X_i(t) \cdot \mu \gamma_i(t) \]  

\[ = \sum_{i=1}^{n} \Delta X_i^2(t) \cdot \mu g \rho \]  

Where \( f_i(t) \) is the friction force that increases with the increase of the length of the catheter, and \( \Delta X_i(t) \) is the actual distance length of each catheter segment, \( \mu \) is the friction coefficient of the catheter surface, \( G_i(t) \) is the gravity of the catheter entering the vascular part, and \( g \) is the gravitational acceleration, \( \rho \) is the linear density of the catheter.

According to formula (3), the following results are obtained, as shown in formula (10).

\[ \sum_{i=1}^{n} \left[ F_i(t) \cdot \Delta X_i(t) - f_i(t) \cdot \Delta x_i(t) \right] \]

\[ \leq \sum_{i=1}^{n} \left[ \frac{E \cdot S}{2X(t)} (\Delta X_i(t) - \Delta x_i(t))^2 + 6 \pi \eta \cdot \Delta x_i(t) + \frac{\Delta X_i^2(t) \cdot \mu g \rho}{X(t)} \right] \]  

D. Mathematical model of comprehensive force and catheter tip force

The mathematical model is shown in formula (11).

\[ F(t) \cdot X(t) - f(t) \cdot x(t) = 6 \pi \eta \cdot X(t)^2 + \frac{E \cdot S}{2X(t)} (X(t) - x(t))^2 \]  

\[ F(t) \cdot X(t) \leq f(t) \cdot x(t) \leq 6 \pi \eta \cdot X(t)^2 + \frac{E \cdot S}{2X(t)} (X(t) - x(t))^2 \]  

Where, \( F(t) \) is the comprehensive force measured in real time from the end clamp, \( f(t) \) is the tip force generated by the collision between the end of the catheter and the blood vessel wall, \( X(t) \) is the actual propulsion distance of the catheter into the blood vessel, and \( x(t) \) is the effective distance of the catheter into the blood vessel.

Because the images obtained during the operation are two-dimensional images, the position information of the catheter tip cannot be obtained accurately. According to a method of image coordinate and spatial coordinate conversion based on the visual assistance of the existing platform proposed by our team, the coordinates of the catheter in the spatial coordinate system can be obtained, that is, the effective distance of the catheter in the blood vessel can be obtained[21].

IV. EXPERIMENTS AND RESULTS

A. Experimental setup

By using the experimental platform of our laboratory, the glass vessel will be used to simulate the real blood vessel. The catheter used in the laboratory is Torcon NB® Advantage Catheter produced by cook company in the United
States ® Advantage catheter, the model of the catheter is 5F, the inner diameter is 1.2mm, the outer diameter is 1.67mm, and the surface friction coefficient of the catheter is 0.18[22]. According to the data, the elastic modulus of the catheter is 8.274Mpa. After measurement, the linear density of the catheter is 0.0036kg/m, and the gravitational acceleration in the experimental area is 9.8N/kg.

During the experiment, the simulated blood vessel is placed flat on the experimental platform, the pressure sensor (YZC131, China) is placed at the outlet of the blood vessel to measure the tip force between the catheter tip and the blood vessel. The real force value of the comprehensive force of the slave side clamp is collected by the load cell (TU-UJ5N, TEAC, Japan). The load cell measures the axial force by detecting the small displacement of the detection shaft when it is pushed and pulled. The experimental platform is shown in Figure 8.

The master operator is operated to push the catheter forward, and the slave operator follows the master action to make corresponding action. Push the catheter through the blood vessel. After consulting the medical literature, the pressure that the vessel wall of patients with cardiovascular disease can bear is less than 0.12N[23]. When it is observed that the value of the pressure sensor placed at the outlet of the blood vessel begins to change within the safety threshold, that is, in the process of changing from 0 to 0.12N, the Arduino Mega microcontroller is used to collect the data information of the load cell in real time, and communicate with the computer through the Arduino microcontroller. Record the data of the catheter moving in the blood vessel for 15 seconds. The force information collected by the load cell is compared with the force information predicted by the mathematical model to verify the effectiveness of the mathematical model.

### B. Experimental results

The comparison diagram is shown in Figure 9, it represents the comprehensive force measured by the load cell and predicted by the mathematical model respectively. The error value is shown in Figure 10, it represents the error between the predicted value of the mathematical model and the actual measured value.

![Fig. 8 Experimental platform](image)

![Fig. 9 Comparison between the true strength value of the slave end catheter and the theoretical strength value of the mathematical model](image)

![Fig. 10 Error diagram between the true strength value of the slave end catheter and the theoretical force value of the mathematical model](image)

### C. Experimental analysis

According to the experiment, when the data of the pressure sensor begins to change, the effective distance $x(t)$ of the catheter advancing forward in the blood vessel was 0.2500m. The actual length $X(t)$ of the catheter in the simulated blood vessel ranges from 0.302 m to 0.3120 m. The compression $\Delta L$ of the catheter ranges from 0.052m to 0.062m, the calculated elastic coefficient $K$ ranges from 29.029N/m to 29.990N/m, and the comprehensive force measured from the end clamp ranges from 0.2079N to 0.4558N. The error is within 20mN, which meets the allowable range of surgical safety[23].

The experimental data showed that when the comprehensive force measured from the slave side was between 0N and 0.4558N, the collision force between the catheter tip and the vascular wall was less than 0.12N, and the operation was within the safe range. When the comprehensive force at the slave side is bigger than 0.4558N, the tip force between the catheter tip and the vascular wall is bigger than 0.12N. At this time, it is not advisable to continue to push the catheter, otherwise the tip of the catheter may damage the blood vessel wall, resulting in the failure of the operation. The method proposed in this paper can not only accurately obtain
the quantitative relationship between the catheter tip force and the comprehensive force of the slave clamp, but also solve the problem that the sensor needs to be disinfected when it entrances the human body.

V. CONCLUSION

In the process of vascular interventional surgery, the complex vascular environment in the human body has become the main reason why it is impossible to accurately measure the tip force between the catheter tip and the vascular wall. Therefore, vascular tissue injury accidents occur from time to time. In this article, a new method is proposed to solve the problem that the force at the tip of the catheter cannot be accurately obtained during the operation. This method uses the mass spring model method to construct the mathematical model of the catheter, and calculates the relationship between the catheter tip force and the comprehensive force based on the law of energy conservation. The experimental results show that this method can calculate the relationship between the catheter tip force and the comprehensive force, and reasonably control the threshold of the comprehensive force of the slave clamp through the catheter tip force. It can also solve the problem that the sensor needs to be disinfected when it enters the human body, and reduce the potential safety hazards of surgery. It is of great significance to ensure the safety of vascular interventional surgery in the future.

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