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Hydrodynamics Simulation of a Three-dimentional Self-propelled Bionic Manta

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***Abstract*—The exploration of the ocean is inseparable from the development of Underwater robots. However, the energy barrier of a single underwater robot is particularly serious, which limits the endurance and application of underwater robots. Some studies believe that the swimming efficiency of fish is higher than that of propeller, which gives inspiration to the propeller design of underwater robot. Therefore, we propose a three-dimensional self-propelled bionic ray fluid structure coupling model, and use Fluent to complete the calculation. The pressure contour and the change of vortex structure in the self propulsion cycle are analyzed, and the movement of high pressure center and the fuse of tail vortex structure is discussed.**

***Index Terms*—bionic manta, hydrodynamic, numerical simulation, Fluid-solid coupling**

I. INTRODUCTION

In recent years, the arduous task of ocean exploration has made people more and more aware of the importance of underwater robots [1]. Therefore, many studies have emerged to make a single underwater robot and hope that it can complete tasks such as marine terrain exploration, fishery resources exploration, military or its types. However, such a single robot is generally characterized by underactuation, hyperredundancy, power distribution, modularization and so on. In view of the functional defects of monomers, some studies have turned to the bionic process of marine organism.

There are many kinds of small prototype for terrestrial creatures, such as rice [2], [3], insects and so on, while prototype for marine creatures is relatively single, which fish species have a place undoubtedly. Meanwhile, some studies believe that the efficiency of traditional propeller propulsion is not as efficient as that of fish swimming [4], [5]. Therefore, various underwater robots were designed by utilizing the bio-prototype where researchers mainly focus on motion control [6], [7],locomotion pattern [8], [9], kinematics [10], [11], material [12], [13], hydrodynamics [14], [15], shape optimization [16].

However, this simple imitation behavior does not clarify the internal mechanism of fish swimming. Therefore, some researchers focus on fish swimming dynamics. The study of fish swimming can be divided into biological observation method and numerical simulation method. Among them, the biological observation method mainly uses the high-speed particle camera to extract the motion characteristics by observing the structure of the wake vortex and the attitude frequency of the motion. Westneathe and Walker use labrid fish to analyse the hydrodynamics which would provide several kinds of data that can be applied to the design of autonomous underwater vehicles [17]. Rosenberger do motion observation of eight kinds of rays and the shape parameters and kinematic parameters are given respectively [18]. Drunker and Lauder use DPIV to discuss the ten "lessons learned" from the application of DPIV to problems of fish locomotion in 1997 [19].

The other is to use numerical simulation. Boranzjani provide the first evidence of the vortex reattachment at the leading edge of the fish tail using three-dimensional high-resolution numerical simulations of self-propelled virtual swimmers with different tail shapes [20]. Li implement a parallel mesh deformation method based on the radial basis function(RBF) interpolation to discuss in detail the efficiency of fish exploiting vortices, the force coefficient and the hydrodynamic interactions between fish and vortices [21]. Zhang focus on the unsteady flow field of undulating fins in stationary water is calculated by Large Eddy Simulation and dynamic grid technique with diffusion-based smoothing model and the vortex characteristic and its temporal and spatial evolution is described, and the vortex structure and its relationship with thrust, heave force is analyzed [22]. Yu use finite-volume method to simulate a fish swimming with the pectoral fins abducted and put an eye on the wake flow structures, forces, and power consumption with respect to various Strouhal numbers [23]. Guo use a novel dynamic-grid generation method, the adaptive control method to calculate the correlation between undulatory locomotion and the flow characteristics of a 2D fish [24].

At present, most of the numerical simulation research on fish movement is to fix the fish in a steady place and observe the changes of pressure contour and vorticity on its surface by means of uniform incoming flow. And by this mean, the kinematic characteristic and the shape optimization are mainly focused on, while the mechanisms of the fish propulsion is ignored. The numerical simulation will help better design the motion mode of the underwater robot fish by studying the periodic movement and scientifically instructed the bionic

TABLE I
SPECIFIC PARAMETER OF THE MANTA RAY MODEL

Characteristic	Size
Length	0.4m
Wingspan	1m
Volume	$4.72 * 10^{-3} m^3$
Surface area	$0.41 m^2$
Centroid	(0.0,-0.15,0.0)

Fig. 1. Manta fish prototype

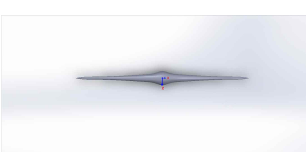
process. Therefore, a three-dimensional self-propelled manta is designed to observe the hydrodynamics and the vortex change to better utilize the energy and generate better efficiency.

II. MODEL SET UP

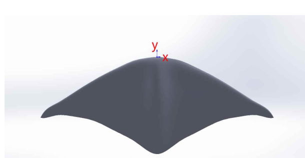
During the long period of evaluation, fish species dominate the marine life, which becomes the bionic prototype of underwater vehicles undoubtedly. Due to the limitation of conditions, we cannot observe organisms. Therefore, we use the shape conditions and swimming characteristics to set up our model.

A. Physical modeling

The Manta ray has a flat body, whose outline of the pectoral fin is similar to a triangle, with a slender tail and a wide pectoral fin. According to Fig.1, the manta ray can be roughly divided into four parts: mouth, pectoral fin, body and tail. In physical modeling, we ignore the tail and mouth. Because the tail does not swing when the ray swims forward in a straight line and its mass relative to the body part is negligible. And the mouthpiece part is located in the center of the axis of symmetry, which has little influence when moving in a straight line. At the same time, its pectoral fins are symmetrically distributed, so only the characteristics of one side need to be considered in physical modeling. We use SolidWorks to model the fish body shape, as shown in Fig.2. And the specific characteristics are shown in table I. The wingspan of the model is 1m, while the body length of the model is 0.4m. The ratio of the length and the span is 0.4, which is consistent with the real manta ray.



(a) top view



(b) front view

Fig. 2. Physical model of the manta ray in two views

B. Kinematic equation set up

According to the experimental observation, only the pectoral fins flap during swimming, and the body can be regarded as a rigid body and the flapping process can be described by wave function.

Establish a coordinate system to describe this motion. Establish global coordinate system (O-$X_w Y_w Z_w$)and fish body coordinate system(o_c-$x_c y_c z_c$). It is defined that the expansion direction of the two pectoral fins is the x_c-axis and the right side of the pectoral fin is the x-positive direction. The connecting line from the fish head to the fish tail is defined as the y_c-axis and the direction of the fish head is the y-positive direction, and the fish head is o_c, the origin of the fish body system. The coordinate system is established by the right-hand rule, and the manta belly direction is the positive direction of z_c-axis. The X_w axis,Y_w axis, Z_w axis of the global coordinate system is parallel with the x_c axis, y_c axis and z_c axis of the fish body coordinate system. The fish body system is defined to describe the fluctuating of the manta.

The traveling waves under the fish body coordinate can be defined as equation(1),

$$z = A(x)sin(\frac{2\pi y}{\lambda} - 2\pi ft + \phi) \tag{1}$$

Where $A(x)$ represents the amplitude control function. λ represents the wavelength, f represents the phase frequency, t represents the time, ϕ represents the initial phase difference.

Aiming at the fish propulsion, the envelope velocity is not considered, for we assume a single frequency of vibration when only a single fish is included in the flow field. Therefore, the phase velocity is related to the vibration frequency of fish. Thus,

$$\omega = 2\pi f \tag{2}$$

Where ω represents the fluctuate frequency of fish. Meanwhile, the amplitude control function $A(x)$ differs from species to species, which means that $A(x)$ is the classification basis of manta fish. And $A(x)$ can be expressed as equation(3),

$$A(x) = c_2 x^2 + c_1 x + c_0 \tag{3}$$

Where c_0, c_1, c_2 is the spanwise amplitude envelope coefficient of pectoral fin. And there are mainly three situations. When c_1= c_2=0, the amplitude envelope function is constant and the amplitude of the spanwise is constant. When c_2=0, the amplitude envelope function is linear and the amplitude of the spanwise is linear variation. When $c_2 \neq 0$, the amplitude envelope function is constant is a quadratic function and the the amplitude of the spanwise present quadratic function change.

We assume that the body length and the total length of wingspan remain unchanged during flapping because of the elastic characteristic of fish connective tissue. Therefore, the kinematic model under the global coordinate is as equation(4),

$$\begin{cases} y(t) = y + dy \\ x(t) = x \\ z(t) = A(x)sin(\frac{2\pi y - dy}{\lambda} - 2\pi ft + \phi) \end{cases} \tag{4}$$

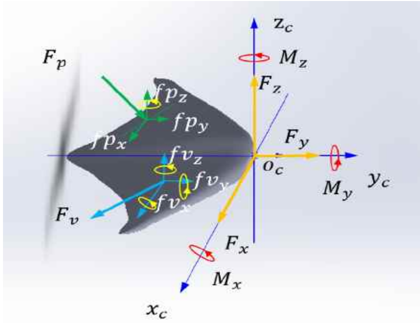


Fig. 3. Forces and momentum sketch under defined coordinate

C. Dynamics modeling

The propulsion of manta fish is mainly dominated by forces from two different sources, the internal driving force and flow field feedback force [25], [26].

1) Force analysis: The internal force is generated by the internal structure, mainly due to the contraction and relaxation of the ray's own muscle, aiming at controlling its stance and speed of swimming automatically, while the external force is generated by the continuous change of flow field in motion, which includes bouyancy, gravity, viscous forces and inertial forces [27]. It is assumed that buoyancy is equal to gravity when the fish is steady to simplify the model.

The inertial force is caused by the pressure on the surface of the contact surface where its direction is perpendicular to the contact surface, and its components in the x_c, y_c, z_c axial directions are fp_x, fp_y and fp_z respectively. While the momentum of components around the x_c, y_c, z_c axis are mp_x, mp_y and mp_z respectively. The viscous force is caused by the viscosity of water, and the direction is parallel to the contact surface, and its components in the x_c, y_c, z_c axial directions are fv_x, fv_y and fv_z respectively. While the momentum of components around the x_c, y_c, z_c axis are mv_x, mv_y and mv_z respectively. And the join forces of the three direction is F_x, F_y, F_z, respectively. And the join momentum around the three axis is M_x, M_y, M_z, respectively. The forces and momentum sketch is as shown in Fig.3. And it is obviously that,

$$\begin{cases} \vec{F_p} + \vec{F_v} = \vec{F} \\ \vec{M_p} + \vec{M_v} = \vec{M} \end{cases} \tag{5}$$

Where F_p represents the force vector caused by pressure and F_v represents the force vector caused by viscous.

2) Dynamic equation set up: According to classical mechanical equation [28], [29], thus,

$$F = m \times \frac{dv}{dt} \tag{6}$$

$$M = J \times \frac{dw}{dt} \tag{7}$$

Where F represents the combined external force of particle system. And M represents the combined external momentum of particle system. v represents the translation speed of the particle system. And w represents the rotation speed of the particle system. m represents the mass of the particle system. And J represents the moment of inertia of the particle system.

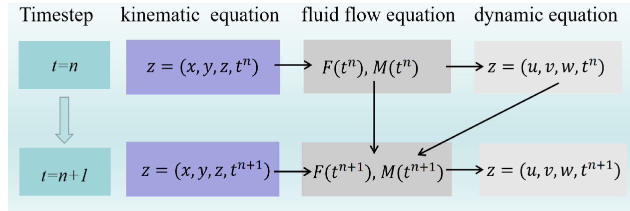


Fig. 4. The sketch of the coupling equation and its solution

According to the equation(5) and (6), we can derived equation(8),

$$F = -\int_{\partial S} (-p\vec{n} + \mu(w \times \vec{n}))ds \tag{8}$$

Where p is the fluid pressure. $\vec{n}$ is the unit vector normal to the body surface of the manta fish. And S is the manta fish surface. μ is the dynamic viscosity of the fluid.

D. Establishment of flow field control equation

In the global coordinate system, the control equation of a flow fluid is as equation(9),

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0 \tag{9}$$

Where ρ represents the density of the fluid and $(u(t), v(t), w(t))$ is the velocity vector component of $\vec{u}$ under the global coordinate system. For water is a continuous incompressible medium, which satisfies the continuity equation(10),

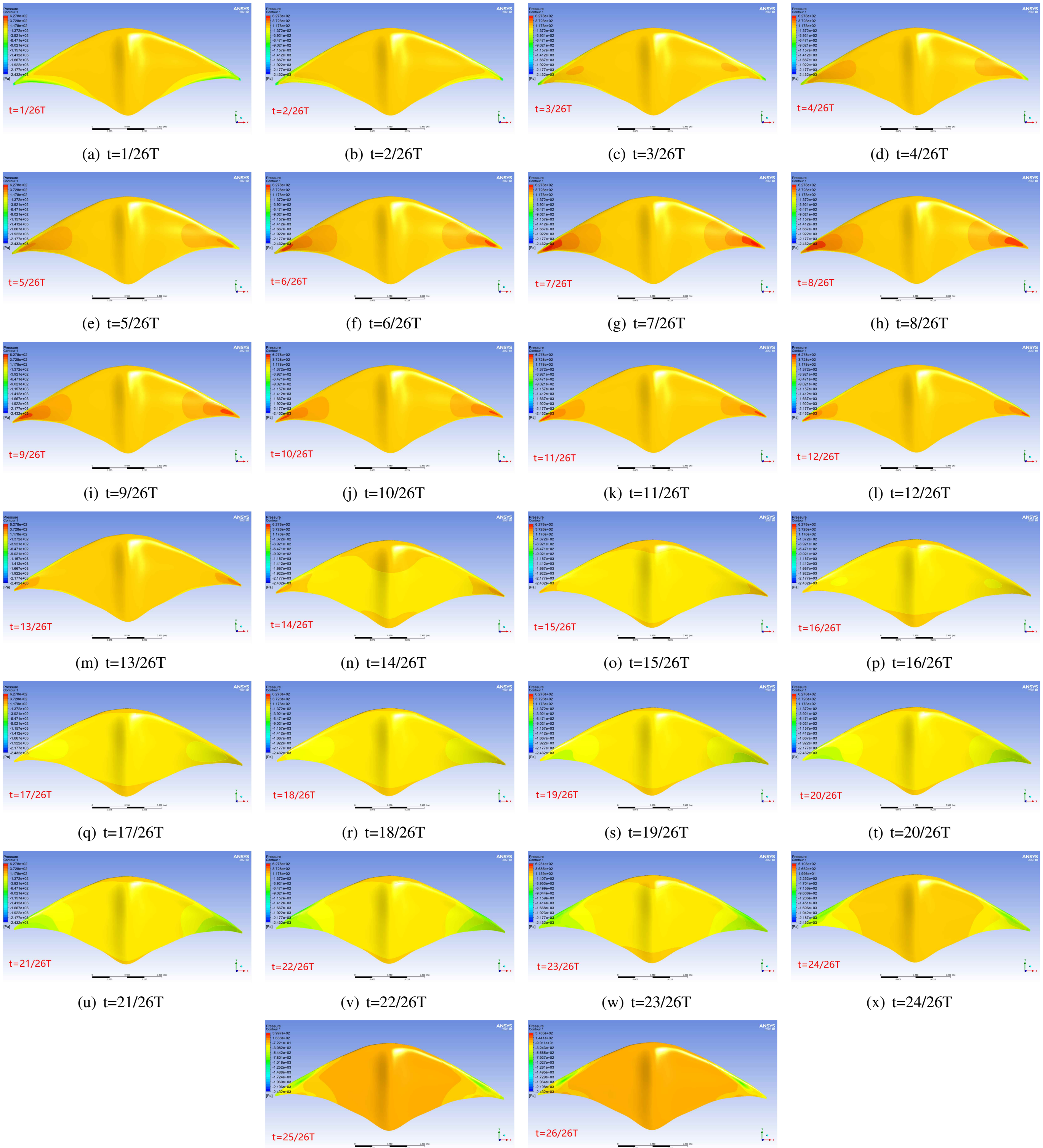
$$\nabla \vec{u} = 0 \tag{10}$$

Using the slip free model,velocity inlet and pressure outlet the following initial and boundary conditions can be obtained as equation(11),

$$\begin{cases} \vec{u}(x_c, y_c, z_c, t)|_{S(t)}^{n} = 0 \\ \vec{u}(x_c, y_c, z_c, t)|_{S(t)} = \vec{u} \\ \vec{u}(x_c, y_c, z_c, t)|_{(t=0)} = u(0) \end{cases} \tag{11}$$

E. Establishment of fluid structure coupling equation

In the initial state, the fish body is stationary and will move according to the fish kinematics equation will lead to the change of the flow field. The change of the flow field will exert external force on the fish body model and change the motion of the fish body at the next timestep and is done alternately until the moving speed reaches the steady-state speed. The kinematic equation is independent and the dynamic equation of the manta and the the flow fluid equation share the same boundary conditions and because of the no-slip model, the force of fluid on fish is equal to that of fish on fluid and the surface velocity distribution is consistent.



(a) t=1/26T (b) t=2/26T (c) t=3/26T (d) t=4/26T

(e) t=5/26T (f) t=6/26T (g) t=7/26T (h) t=8/26T

(i) t=9/26T (j) t=10/26T (k) t=11/26T (l) t=12/26T

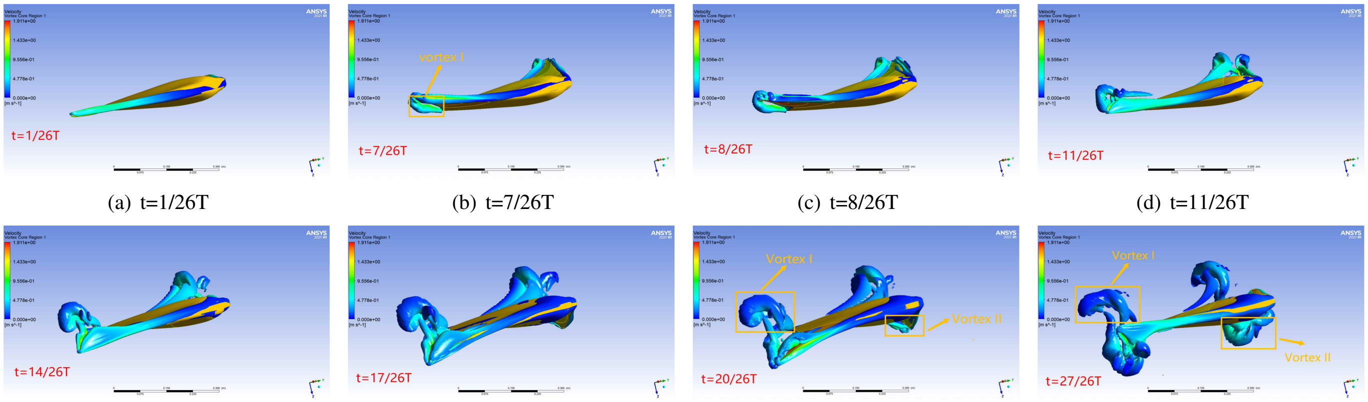
(m) t=13/26T (n) t=14/26T (o) t=15/26T (p) t=16/26T

(q) t=17/26T (r) t=18/26T (s) t=19/26T (t) t=20/26T

(u) t=21/26T (v) t=22/26T (w) t=23/26T (x) t=24/26T

(y) t=25/26T (z) t=26/26T

Fig. 5. Pressure contour during one self-propelled cycle



(a) t=1/26T (b) t=7/26T (c) t=8/26T (d) t=11/26T
(e) t=14/26T (f) t=17/26T (g) t=20/26T (h) t=27/26T

Fig. 6. Vortex structure during the first self-propelled cycle

III. Simulation and Experiments

Previous studies have been published in conference papers and we mainly focus on the self-propelled process of the manta in this paper [30]. The amplitude control function is $A(x) = 0.5x^2 - 0.1x + 0.005$ and the kinematic equation is $z = A(x)sin(\frac{2\pi y}{20} - 12.5t)$.One cycle is around 0.52s and the timestep is 0.0008s. The initial position of the manta fish is when it is tilling.

A. Pressure analysis

According to the analysis of the equation, during the first quarter of the swimming cycle the fin flap downward to the limit position on $-z$ direction, while the second quarter of the cycle, the fin flap upward to the initial position and the third quarter of the swimming period the fin flap upward to the limit position on z direction. Then, the last quarter of the cycle, the fin flap downwards. One self-propelled cycle of manta fish is simulated and the pressure contour is shown in Fig.5. Fig.5(a)-(g) shows the fin flap upward and is the first quarter of the cycle. And Fig.5(g) is the limit position in direction $-z$. Fig.5(h)-(m) shows the fin flap downward and is the second quarter of the cycle. And Fig.5(m) is the initial position. The Fig.5(n)-(t), shows the fin flap upward and is the third quarter of the cycle. And Fig.5(t) is the limit position in direction z. Fig.5(u)-(z) shows the fin flap downward and is the last quarter of the cycle. And Fig.5(z) is the initial position.

According to the pressure contour, it is easy to conclude that during the downward period, there are two high-pressure center formed from both fins and move from the front of the moving direction to the back of the moving direction. And the high-pressure center can be observed on the upstream surface of the fish body. Therefore, we can conduct that two high-pressure centers will form on the surface of the downstream surface, which we can not observe directly on both fins during the period of flapping upward. And that means four high-pressure center will form during one cycle, which will cause the flow fluid change and therefore, the fish body produces a pressure difference in the y direction, so that the manta fish body can achieve self-propelled.

When the manta fish flapping in the positive direction of the z-axis as shown in Fig.5(h)-(t), the upstream surface of its pectoral fin generates positive pressure, the downstream surface generates negative pressure, and the force generated at the farthest end of its pectoral fin is the largest. And the pressure distribution area shows that the thrust is the largest when the fish is at the limit position, which represents the highest generated force happens when the upward period is terminate or the downward period is terminate.

The head of the fish is always under high pressure and that is consistent with drag on true condition of fish swimming. And the moving direction is the same with y-axis and the displacement is not obvious in the first quarter of the cycle because there is a certain hysteresis from the static state to the swimming state of the fish.

B. Vortex contour analysis

The vortex change during the first cycle of self-propelled is shown in Fig.6. Fig.6(b) is the limit position on direction $-z$ and Fig.6(g) is the limit position on direction z. Fig.6(c)-(g) can be recognized as the downward process of the flapping process. Fig.6(a), (b) shows that vortex I form during the upward process of fin and is located at the edge of the downstream fin. Fig.6(g), (h) shows the upward process and Voretex II is formed at the beginning of the turning moment, which also means the fins reach the limit position, a new vortex is generated. The water vortices generated by the upward and downward flapping of the pectoral fin are opposite and rotate up and down staggered [31].

It can be concluded that in the upward process, two vortex will fall off from the two fins, and in the downward process, two opposite vortex will fall off from two fins, and over time, the two water vortices will merge together to form a tail vortex. And the vortex are basically symmetrical, which reflects that the force on the fish body tends to be zero in the X direction.

IV. Conclusion

To conclude, a self-propelled model of the bionic manta ray is purposed and simulated through ANSYS Fluent aiming at the mechanism of the manta fish propulsion process.

It is easy to observe the high-pressure center form and move during the self-propelled process through the pressure contour that every cycle will produce four high-pressure center and the pressure distribution is the most when the fin is posed in limit position. The move of the pressure center makes the pressure difference in different time, which is the thrust of fish swimming forward and can be observed in the displacement of the manta fish.

And the vortex of the swimming period show that the vortex drop off from the fin have different direction when the fin is upward or downward. And the vortex will move downstream and different vortices stagger up and down and fuse together at last.

The future work will focus on the schooling effect of the manta ray to carry out energy-saving strategies for underwater bionic robot. And the regular pattern of the energy transition in the vortex change with time needs to be further discussed to better utilize the energy evoked by an underwater individual and improve the overall performance of the underwater robot system.

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